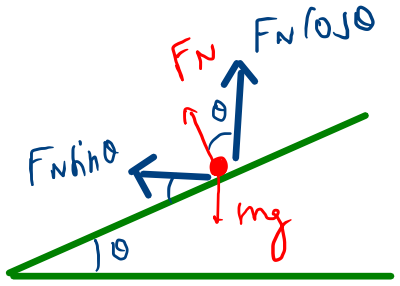
Banking angle. (*a*) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required. (*b*) What is this angle for a road which has a curve of radius 50 m with a design speed of 50 km/h?



$$F_N \sin\theta = \frac{mv^2}{r}$$

$$F_N \cos\theta = mg$$

$$\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\Rightarrow \tan\theta = \frac{mv^2}{r} \cdot \frac{1}{mg}$$

$$\Rightarrow \boxed{\tan\theta = \frac{v^2}{gr}}$$

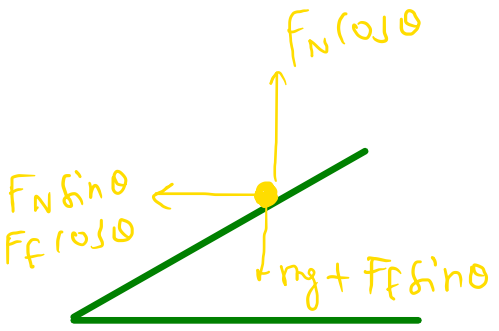
$$50\,km/h = \frac{50 \times 1000}{3600}$$

$$= 13.8888\,m/s$$

b) $$\tan\theta = \frac{(13.8888)^2}{(9.8)(50)}$$

$$\Rightarrow \boxed{\theta \approx 21.49^\circ}$$

(III) If a curve with a radius of 95 m is properly banked for a car traveling 65 km/h, what must be the coefficient of static friction for a car not to skid when traveling at 95 km/h?



$$F_N\sin\theta + F_f\cos\theta = \frac{mv^2}{r}$$

$$\Rightarrow F_N\sin\theta + \mu F_N\cos\theta = \frac{mv^2}{r}$$

$$\Rightarrow \boxed{F_N\left(\sin\theta + \mu\cos\theta\right) = \frac{mv^2}{r}} \quad -(2)$$

$$\frac{F_N(\cos\theta - \mu\sin\theta)}{F_N(\sin\theta + \mu\cos\theta)} = \frac{mg}{\frac{mv^2}{r}}$$

$$\Rightarrow \frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta} = \frac{gr}{v^2}$$

$$\Rightarrow \frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta} = \frac{(9.8)(95)}{(26.3888)^2}$$

$$\Rightarrow \boxed{\mu \approx 0.315}$$

$$\boxed{\theta \approx 19.2985^\circ}$$

$95\,km/h = \frac{95\times 1000}{3600} \approx 26.3888\ m/s$

(III) A curve of radius 78 m is banked for a design speed of 85 km/h. If the coefficient of static friction is 0.30 (wet pavement), at what range of speeds can a car safely make the curve? [*Hint*: Consider the direction of the friction force when the car goes too slow or too fast.]

$$\frac{mg}{\cos\theta + \mu\sin\theta} = \frac{mv^2}{r(\sin\theta - \mu\cos\theta)}$$

$v_{upper} = 31.73 \text{ m/s}$

$16.41 \leq v \leq 31.73$

$$\Rightarrow \frac{gr(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta} = v^2$$

$$\Rightarrow v^2 = \frac{(9.8)(78)(\sin 36.1 - 0.3\cos 36.1)}{\cos 36.1 + 0.3\sin 36.1} = \frac{265.09}{0.9847}$$

$\Rightarrow v = 16.41 \text{ m/s}$

$= 23.61 \text{ m/s}$

$\theta = 36.10^\circ$

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