x	-4	-3	-2	-1	0	1	2	3	4	8	9
f(x)	0	1	3	-5	-1	7	-3	5	2	-2	-6

Selected values of the continuous function f(x) are shown in the table above. Use the values in the table to answer the following.

1. Let g(x) = 3f(x+2) - 1. (a) Find g(1). (b) Find g(-2). (c) If g(k) = -7, find k.

2. Let $h(x) = 5 - f(2x)$.		
(a) Find $h(2)$.	(b) Find $h(0)$.	(c) Find $h^{-1}(4)$.

3. Let p(x) be the function that results from applying three transformations to the graph of f in this order: a horizontal dilation by a factor of 3, a reflection over the x axis, and a vertical translation by -4 units.

(a) Find <i>p</i> (3).	(b) Find $p(-6)$.	(c) If $p(x) = f(x)$, find x.
	(b) f in $a p(-b)$.	

4. Let m(x) = af(bx) + c, where *a*, *b*, and *c* are positive constants. The graph of *m* can be constructed by applying three transformations to the graph of *f* in this order: a horizontal dilation by a factor of $\frac{1}{2}$, a vertical dilation by a factor of $\frac{1}{2}$, and a vertical translation by 3 units. (a) Find m(-2). (b) Find m(4). (c) If m(k) = 0, find *k*.

x	-3	-1	0	1	3	4	6	9
g(x)	-4	2	3	6	1	-1	-5	-2

Selected values of the continuous function g(x) are shown in the table above. Use the values in the table to answer the following.

5. Let h(x) = -2g(x-3) - 5. (a) Find h(0). (b) Find h(3). (c) If h(k) = 5, find k.

6. Let
$$n(x) = 2 + g\left(\frac{x}{3}\right)$$
.
(a) Find $n(3)$.
(b) Find $n(-3)$.
(c) Find $n^{-1}(4)$.

7. Let p(x) be the function that results from applying three transformations to the graph of g in this order:a horizontal dilation by a factor of $\frac{1}{2}$, a reflection over the y axis, and a vertical translation by 1 unit.(a) Find p(-2).(b) Find the average rate of change of p over the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

8. Let s(x) = ag(bx) + c, where *a*, *b*, and *c* are positive constants. The graph of *s* can be constructed by applying three transformations to the graph of *g* in this order: a horizontal dilation by a factor of 3, a vertical dilation by a factor of 4, and a vertical translation by -5 units.

(a) Find s(3). (b) Find s(-9). (c) If s(k) = -9, find k.

x	-5	-2	-1	2	3	4	6	12	15
h(x)	6	1	0	-3	-2	2	8	11	9

Selected values of the continuous function h(x) are shown in the table above. Use the values in the table to answer the following.

- 9. Let h(x) = 6f(x+2) 3.
- (a) Find f(4). (b) Find f(0).

(c) If f(k) = 2, find *k*.

10. Let $h(x) = -2g\left(\frac{x}{2}\right)$. (a) Find g(6). (b) If g(x) = 1, find x.

(c) Put the following in order from least to greatest: g(-1), g(1), g(2).

11. Let h(x) be the function that results from applying three transformations to the graph of j in this order: a horizontal dilation by a factor of $\frac{1}{3}$, a vertical dilation by a factor of 2, and a vertical translation by -4 units. (a) Find j(6). (b) Find j(-3).

12. Let w(x) = 2h(x-3) + 1(a) Find $w(-2) \cdot h(6)$. (b) Find w(h(-5)). (c) Find w(w(2)).



The graph of f(x) is shown in the figure above and consists of three line segments. 13. Let g(x) = -2f(x + 1). Sketch the graph of g(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three line segments. 14. Let h(x) = f(2x) - 3. Sketch the graph of h(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three line segments. 15. Let f(x) = 2k(x - 2) - 1. Sketch the graph of k(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three line segments. 16. Let g(x) = 1 - f(2x). Sketch the graph of g(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three line segments. 17. Let h(x) = f(x - 2) + 3. Sketch the graph of h(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three line segments. 18. Let f(x) = 2k(x + 1). Sketch the graph of k(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three linear pieces and a point at (4, 3). 19. Let g(x) = 2f(x + 3) - 1. Sketch the graph of g(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three linear pieces and a point at (4, 3). 20. Let h(x) = f(2x - 2) - 4. Sketch the graph of h(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of three linear pieces and a point at (4, 3).

21. Let k(x) be the function that results from applying three transformations to the graph of f in this order: a vertical dilation by a factor of 2, a reflection over the y axis, and a vertical translation by -1 unit. Sketch the graph of k(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of two linear pieces, a semi-circle, and a point at (2, -1). 22. Let g(x) = -f(2x). Sketch the graph of g(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of two linear pieces, a semi-circle, and a point at (2, -1). 23. Let h(x) = f(-x) + 2. Sketch the graph of h(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and consists of two linear pieces, a semi-circle, and a point at (2, -1). 24. Let k(x) be the function that results from applying three transformations to the graph of f in this order: a horizontal dilation by a factor of $\frac{1}{2}$, a reflection over the x axis, and a vertical translation by 2 units. Sketch the graph of k(x) on the same axes as f(x) above.

Worksheet: (Topic 1.12)

Transformations of Functions



The graph of f(x) is shown in the figure above and has the domain [-3, 3] and the range [-5, 3]. 25. Let g(x) = f(x + 2) - 1. Sketch the graph of g(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and has the domain [-3, 3] and the range [-5, 3]. 26. Let $h(x) = 1 - f\left(\frac{x}{2}\right)$. Sketch the graph of h(x) on the same axes as f(x) above.



The graph of f(x) is shown in the figure above and has the domain [-3, 3] and the range [-5, 3].

27. Let k(x) = -3f(2x) + 1.

Find the domain and range of k(x).

28. Let $p(x) = \frac{1}{2}f(x+3) - 4$. Find the domain and range of p(x).

Transformations of Functions

29. The graph of f(x) has zeros at x = -2, 0, and 3. Find the zeros of the following functions.

(a)
$$g(x) = 2f(x-4)$$
 (b) $h(x) = -\frac{1}{3}f(2x)$ (c) $k(x) = -5f(3x-2)$

30. The graph of f(x) has the vertical asymptote x = -2 and horizontal asymptote y = 3. Find the vertical and horizontal asymptotes of the following functions.

(a)
$$g(x) = 2f(x+1) - 3$$
 (b) $h(x) = 4 - 3f\left(\frac{x}{5}\right)$ (c) $k(x) = \frac{1}{2}f(4 - 2x) + 3$

31. The graph of f(x) is continuous where $\lim_{x \to -\infty} f(x) = 4$ and $\lim_{x \to \infty} f(x) = -\infty$. (a) If g(x) = -2f(x+7) + 5, find $\lim_{x \to -\infty} g(x)$ and $\lim_{x \to \infty} g(x)$.

(b) If
$$h(x) = -f(-x)$$
, find $\lim_{x \to -\infty} h(x)$ and $\lim_{x \to \infty} h(x)$.

32. The graph of f(x) has the vertical asymptote x = 5 and horizontal asymptote y = -3. Find the vertical and horizontal asymptotes of the following functions that result from transforming the graph of f.

(a) The graph of g results from applying the following transformations to graph of f in this order: vertical dilation by a factor of 3, reflection over the x axis, reflection over the y axis, and a horizontal translation by - 3 units.

(b) The graph of h results from applying the following transformations to graph of f in this order: horizontal dilation by a factor of 2 and a horizontal translation by 4 units.

33. Let $f(x) = x^2 + 4x + 1$. Write an equation of the following functions that are transformations of f(x). (a) g(x) = 3f(x) - 7 (b) h(x) = -2f(x) + 2 (c) k(x) = f(x - 2)

(d)
$$m(x) = f\left(\frac{x}{2}\right) + 3$$
 (e) $p(x) = 2f(x+1) - 5$ (f) $s(x) = -f(-x)$

34. Let $f(x) = \frac{x-1}{(x+2)(x-3)}$. Write an equation of the following functions that are transformations of f(x). (a) g(x) = f(x+4) (b) $h(x) = -2f\left(\frac{x}{3}\right)$ (c) k(x) = f(4-x)

35. Let $f(x) = 2x^2 - 3$. Write an equation of the following functions that are transformations of f(x). (a) g(x) = f(2x - 3) + 2 (b) h(x) = 4f(x) + 1

(c) k(x) results when the graph of f has a horizontal dilation by a factor of 3, followed by a horizontal translation by -5 units, and a vertical translation by 2 units.