# AP Physics

# Review Packet

# ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

## CONSTANTS AND CONVERSION FACTORS

| | | | |
|---|---|---|---|
| Proton mass, | $m_p = 1.67 \times 10^{-27}$ kg | Electron charge magnitude, | $e = 1.60 \times 10^{-19}$ C |
| Neutron mass, | $m_n = 1.67 \times 10^{-27}$ kg | 1 electron volt, | $1 \text{ eV} = 1.60 \times 10^{-19}$ J |
| Electron mass, | $m_e = 9.11 \times 10^{-31}$ kg | Speed of light, | $c = 3.00 \times 10^{8}$ m/s |
| Avogadro's number, | $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$ | Universal gravitational constant, | $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$ |
| Universal gas constant, | $R = 8.31$ J/(mol•K) | Acceleration due to gravity at Earth's surface, | $g = 9.8\ \text{m/s}^2$ |
| Boltzmann's constant, | $k_B = 1.38 \times 10^{-23}$ J/K | | |

| | |
|---|---|
| 1 unified atomic mass unit, | $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$ |
| Planck's constant, | $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$ |
| | $hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$ |
| Vacuum permittivity, | $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$ |
| Coulomb's law constant, | $k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$ |
| Vacuum permeability, | $\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$ |
| Magnetic constant, | $k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$ |
| 1 atmosphere pressure, | $1 \text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa |

| UNIT SYMBOLS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | meter, | m | mole, | mol | watt, | W | farad, | F |
| | kilogram, | kg | hertz, | Hz | coulomb, | C | tesla, | T |
| | second, | s | newton, | N | volt, | V | degree Celsius, | °C |
| | ampere, | A | pascal, | Pa | ohm, | Ω | electron volt, | eV |
| | kelvin, | K | joule, | J | henry, | H | | |

| PREFIXES | | |
|---|---|---|
| Factor | Prefix | Symbol |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | μ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

| $\theta$ | 0° | 30° | 37° | 45° | 53° | 60° | 90° |
|---|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $1/2$ | $3/5$ | $\sqrt{2}/2$ | $4/5$ | $\sqrt{3}/2$ | 1 |
| $\cos\theta$ | 1 | $\sqrt{3}/2$ | $4/5$ | $\sqrt{2}/2$ | $3/5$ | $1/2$ | 0 |
| $\tan\theta$ | 0 | $\sqrt{3}/3$ | $3/4$ | 1 | $4/3$ | $\sqrt{3}$ | $\infty$ |

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

# ADVANCED PLACEMENT PHYSICS C EQUATIONS

## MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_N\right|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

$a$ = acceleration
$E$ = energy
$F$ = force
$f$ = frequency
$h$ = height
$I$ = rotational inertia
$J$ = impulse
$K$ = kinetic energy
$k$ = spring constant
$\ell$ = length
$L$ = angular momentum
$m$ = mass
$P$ = power
$p$ = momentum
$r$ = radius or distance
$T$ = period
$t$ = time
$U$ = potential energy
$v$ = velocity or speed
$W$ = work done on a system
$x$ = position
$\mu$ = coefficient of friction
$\theta$ = angle
$\tau$ = torque
$\omega$ = angular speed
$\alpha$ = angular acceleration
$\phi$ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_G\right| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

## ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0} \left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

$A$ = area
$B$ = magnetic field
$C$ = capacitance
$d$ = distance
$E$ = electric field
$\mathcal{E}$ = emf
$F$ = force
$I$ = current
$J$ = current density
$L$ = inductance
$\ell$ = length
$n$ = number of loops of wire per unit length
$N$ = number of charge carriers per unit volume
$P$ = power
$Q$ = charge
$q$ = point charge
$R$ = resistance
$r$ = radius or distance
$t$ = time
$U$ = potential or stored energy
$V$ = electric potential
$v$ = velocity or speed
$\rho$ = resistivity
$\Phi$ = flux
$\kappa$ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

# ADVANCED PLACEMENT PHYSICS C EQUATIONS

## GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

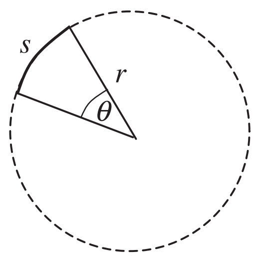
$a^2 + b^2 = c^2$

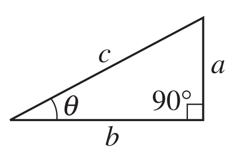
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

$A$ = area
$C$ = circumference
$V$ = volume
$S$ = surface area
$b$ = base
$h$ = height
$\ell$ = length
$w$ = width
$r$ = radius
$s$ = arc length
$\theta$ = angle




## CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

## VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

**PHYSICS C**
**Section II, MECHANICS**
**Time—45 minutes**
**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.

Mech 1.

A crash test car of mass 1,000 kg moving at constant speed of 12 m/s collides completely inelastically with an object of mass $M$ at time $t = 0$. The object was initially at rest. The speed $v$ in m/s of the car-object system after the collision is given as a function of time $t$ in seconds by the expression

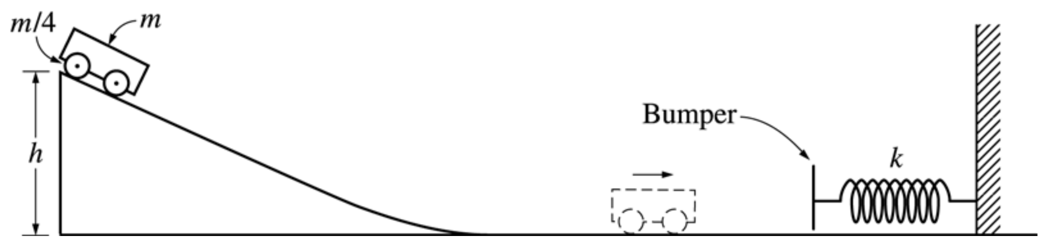
$$v = \frac{8}{1 + 5t}.$$

(a) Calculate the mass $M$ of the object.

(b) Assuming an initial position of $x = 0$, determine an expression for the position of the car-object system after the collision as a function of time $t$.

(c) Determine an expression for the resisting force on the car-object system after the collision as a function of time $t$.

(d) Determine the impulse delivered to the car-object system from $t = 0$ to $t = 2.0$ s.




Mech 2.

The cart shown above is made of a block of mass $m$ and four solid rubber tires each of mass $m/4$ and radius $r$. Each tire may be considered to be a disk. (A disk has rotational inertia $\frac{1}{2}ML^2$, where $M$ is the mass and $L$ is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height $h$. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the total rotational inertia of all four tires.

(b) Determine the speed of the cart when it reaches the bottom of the incline.

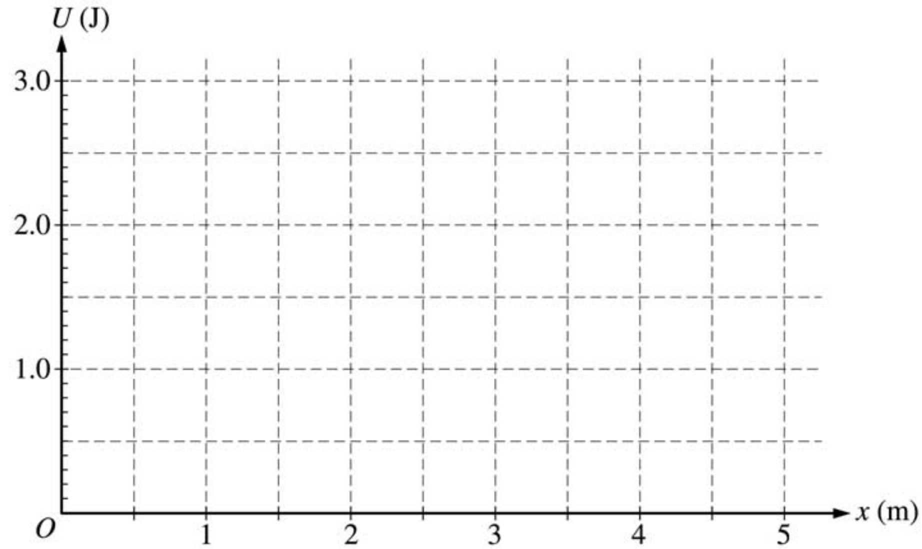
(c) After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant $k$. Determine the distance $x_m$ the spring is compressed before the cart and bumper come to rest.

(d) Now assume that the bumper has a non-neglible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of $x_m$ in part (c). Give a reasonable explanation for this decrease.

Mech 3.

An object of mass 0.5 kg experiences a force that is associated with the potential energy function $U(x) = \dfrac{4.0}{2.0 + x}$, where $U$ is in joules and $x$ is in meters.

(a) On the axes below, sketch the graph of $U(x)$ versus $x$.




(b) Determine the force associated with the potential energy function given above.

(c) Suppose that the object is released from rest at the origin. Determine the speed of the particle at $x = 2$ m.

In the laboratory, you are given a glider of mass 0.5 kg on an air track. The glider is acted on by the force determined in part (b). Your goal is to determine experimentally the validity of your theoretical calculation in part (c).

(d) From the list below, select the additional equipment you will need from the laboratory to do your experiment by checking the line next to each item. If you need more than one of an item, place the number you need on the line.

___ Meterstick ___ Stopwatch ___ Photogate timer ___ String ___ Spring

___ Balance ___ Wood block ___ Set of objects of different masses

(e) Briefly outline the procedure you will use, being explicit about what measurements you need to make in order to determine the speed. You may include a labeled diagram of your setup if it will clarify your procedure.

**END OF SECTION II, MECHANICS**

**PHYSICS C**

**Section II, MECHANICS**

**Time—45 minutes**

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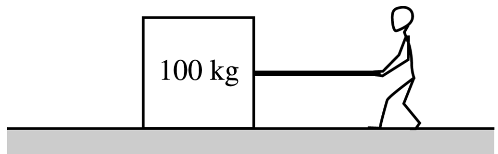



Mech. 1.

The 100 kg box shown above is being pulled along the $x$-axis by a student. The box slides across a rough surface, and its position $x$ varies with time $t$ according to the equation $x = 0.5t^3 + 2t$, where $x$ is in meters and $t$ is in seconds.

(a) Determine the speed of the box at time $t = 0$.

(b) Determine the following as functions of time $t$.

   i. The kinetic energy of the box

   ii. The net force acting on the box

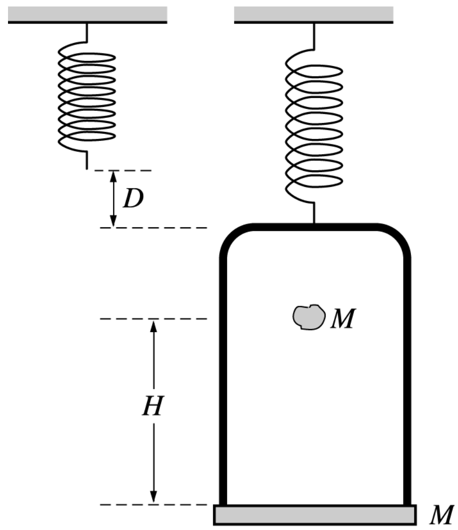
   iii. The power being delivered to the box

(c) Calculate the net work done on the box in the interval $t = 0$ to $t = 2$ s.

(d) Indicate below whether the work done on the box by the student in the interval $t = 0$ to $t = 2$ s would be greater than, less than, or equal to the answer in part (c).

____Greater than ____Less than ____Equal to

Justify your answer.




Mech. 2.

An ideal spring is hung from the ceiling and a pan of mass $M$ is suspended from the end of the spring, stretching it a distance $D$ as shown above. A piece of clay, also of mass $M$, is then dropped from a height $H$ onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the speed of the clay at the instant it hits the pan.

(b) Determine the speed of the pan just after the clay strikes it.

(c) Determine the period of the simple harmonic motion that ensues.

(d) Determine the distance the spring is stretched (from its initial unstretched length) at the moment the speed of the pan is a maximum. Justify your answer.

(e) The clay is now removed from the pan and the pan is returned to equilibrium at the end of the spring. A rubber ball, also of mass $M$, is dropped from the same height $H$ onto the pan, and after the collision is caught in midair before hitting anything else.

Indicate below whether the period of the resulting simple harmonic motion of the pan is greater than, less than, or the same as it was in part (c).

_____ Greater than _____ Less than _____ The same as

Justify your answer.

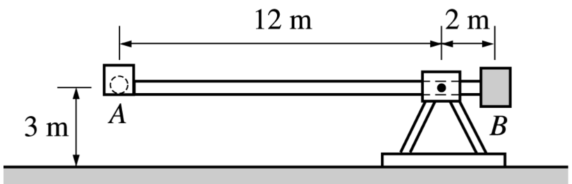



Figure 1

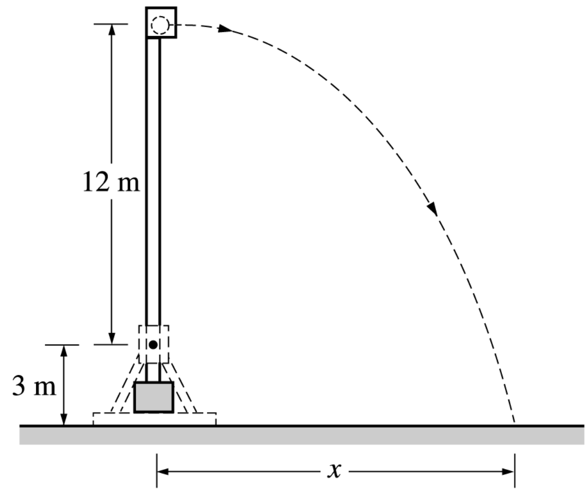



Figure 2

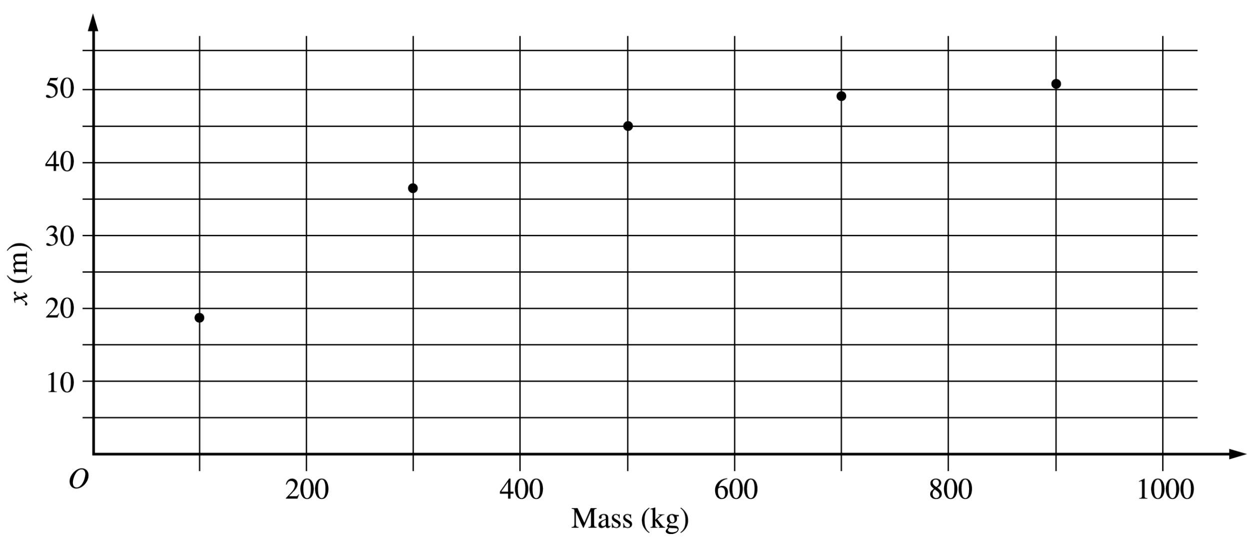
Mech. 3.

Some physics students build a catapult, as shown above. The supporting platform is fixed firmly to the ground. The projectile, of mass 10 kg, is placed in cup $A$ at one end of the rotating arm. A counterweight bucket $B$ that is to be loaded with various masses greater than 10 kg is located at the other end of the arm. The arm is released from the horizontal position, shown in Figure 1, and begins rotating. There is a mechanism (not shown) that stops the arm in the vertical position, allowing the projectile to be launched with a horizontal velocity as shown in Figure 2.

(a) The students load five different masses in the counterweight bucket, release the catapult, and measure the resulting distance $x$ traveled by the 10 kg projectile, recording the following data.

| Mass (kg) | 100 | 300 | 500 | 700 | 900 |
|---|---|---|---|---|---|
| $x$ (m) | 18 | 37 | 45 | 48 | 51 |

i. The data are plotted on the axes below. Sketch a best-fit curve for these data points.




ii. Using your best-fit curve, determine the distance $x$ traveled by the projectile if 250 kg is placed in the counterweight bucket.

(b) The students assume that the mass of the rotating arm, the cup, and the counterweight bucket can be neglected. With this assumption, they develop a theoretical model for $x$ as a function of the counterweight mass using the relationship $x = v_x t$, where $v_x$ is the horizontal velocity of the projectile as it leaves the cup and $t$ is the time after launch.

i. How many seconds after leaving the cup will the projectile strike the ground?

ii. Derive the equation that describes the gravitational potential energy of the system relative to the ground when in the position shown in Figure 1, assuming the mass in the counterweight bucket is $M$.

iii. Derive the equation for the velocity of the projectile as it leaves the cup, as shown in Figure 2.

(c)

i. Complete the theoretical model by writing the relationship for $x$ as a function of the counterweight mass using the results from (b)i and (b)iii.

ii. Compare the experimental and theoretical values of $x$ for a counterweight bucket mass of 300 kg. Offer a reason for any difference.

**END OF SECTION II, MECHANICS**

**PHYSICS C**
**Section II, MECHANICS**
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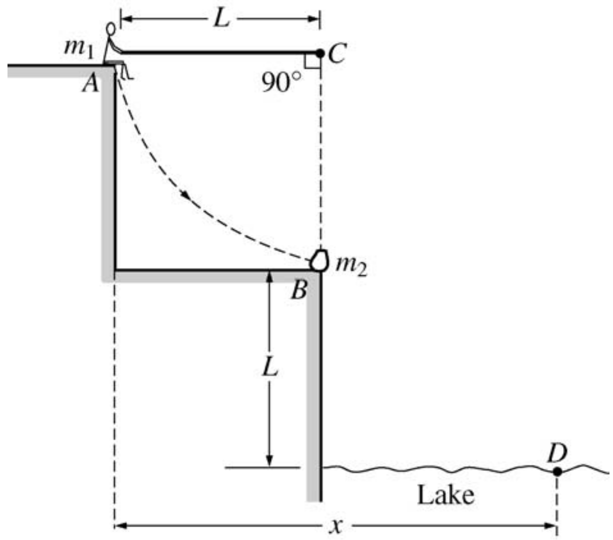



Mech. 1.

A rope of length $L$ is attached to a support at point $C$. A person of mass $m_1$ sits on a ledge at position $A$ holding the other end of the rope so that it is horizontal and taut, as shown above. The person then drops off the ledge and swings down on the rope toward position $B$ on a lower ledge where an object of mass $m_2$ is at rest. At position $B$ the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point $D$, which is a vertical distance $L$ below position $B$. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of $m_1$, $m_2$, $L$, and $g$.

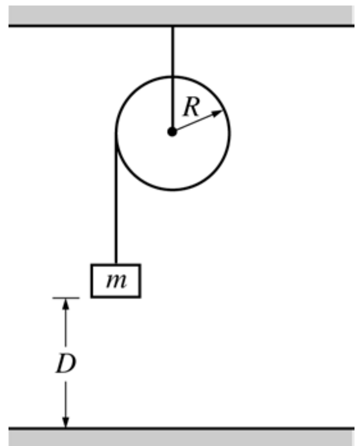
(a) The speed of the person just before the collision with the object

(b) The tension in the rope just before the collision with the object

(c) The speed of the person and object just after the collision

(d) The ratio of the kinetic energy of the person-object system before the collision to the kinetic energy after the collision

(e) The total horizontal displacement $x$ of the person from position $A$ until the person and object land in the water at point $D$.




Mech. 2.

A solid disk of unknown mass and known radius $R$ is used as a pulley in a lab experiment, as shown above. A small block of mass $m$ is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass $m$ is released from rest and takes a time $t$ to fall the distance $D$ to the floor.

(a) Calculate the linear acceleration $a$ of the falling block in terms of the given quantities.

(b) The time $t$ is measured for various heights $D$ and the data are recorded in the following table.

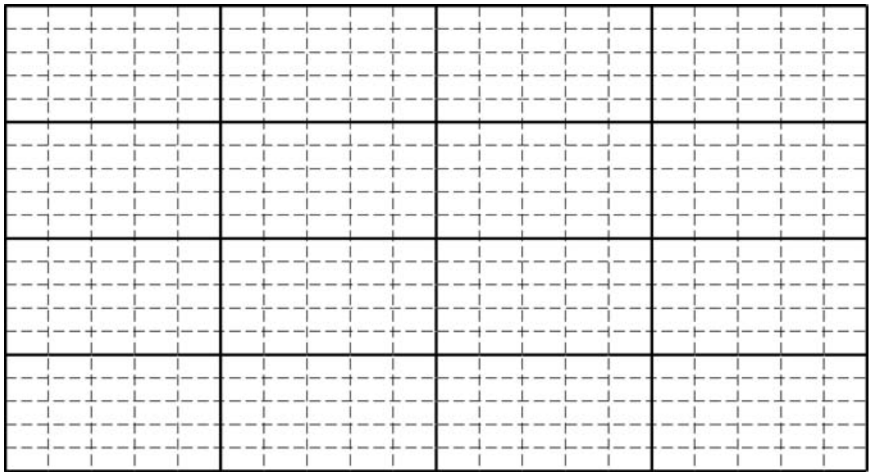
| $D$ (m) | $t$ (s) |
|---|---|
| 0.5 | 0.68 |
| 1 | 1.02 |
| 1.5 | 1.19 |
| 2 | 1.38 |

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.



GO ON TO THE NEXT PAGE.

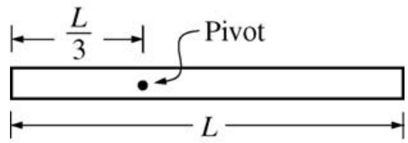
ii. On the grid below, plot the quantities determined in (b)i., label the axes, and draw the best-fit line to the data.



iii. Use your graph to calculate the magnitude of the acceleration.

(c) Calculate the rotational inertia of the pulley in terms of $m$, $R$, $a$, and fundamental constants.

(d) The value of acceleration found in (b)iii, along with numerical values for the given quantities and your answer to (c), can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

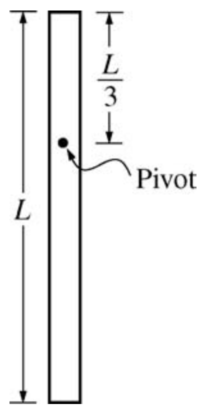



Mech. 3.

A uniform rod of mass $M$ and length $L$ is attached to a pivot of negligible friction as shown above. The pivot is located at a distance $\frac{L}{3}$ from the left end of the rod. Express all answers in terms of the given quantities and fundamental constants.

(a) Calculate the rotational inertia of the rod about the pivot.

(b) The rod is then released from rest from the horizontal position shown above. Calculate the linear speed of the bottom end of the rod when the rod passes through the vertical.




(c) The rod is brought to rest in the vertical position shown above and hangs freely. It is then displaced slightly from this position. Calculate the period of oscillation as it swings.

**END OF SECTION II, MECHANICS**

## PHYSICS C
### Section II, MECHANICS
### Time—45 minutes
### 3 Questions

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.

Mech. 1.

A ball of mass $M$ is thrown vertically upward with an initial speed of $v_0$. It experiences a force of air resistance given by $\mathbf{F} = -k\mathbf{v}$, where $k$ is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of $M$, $k$, $v_0$, and fundamental constants.

(a) Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

____ increases ____ decreases ____ remains the same

Justify your answer.

(b) Write, but do NOT solve, a differential equation for the instantaneous speed $v$ of the ball in terms of time $t$ as the ball moves upward.

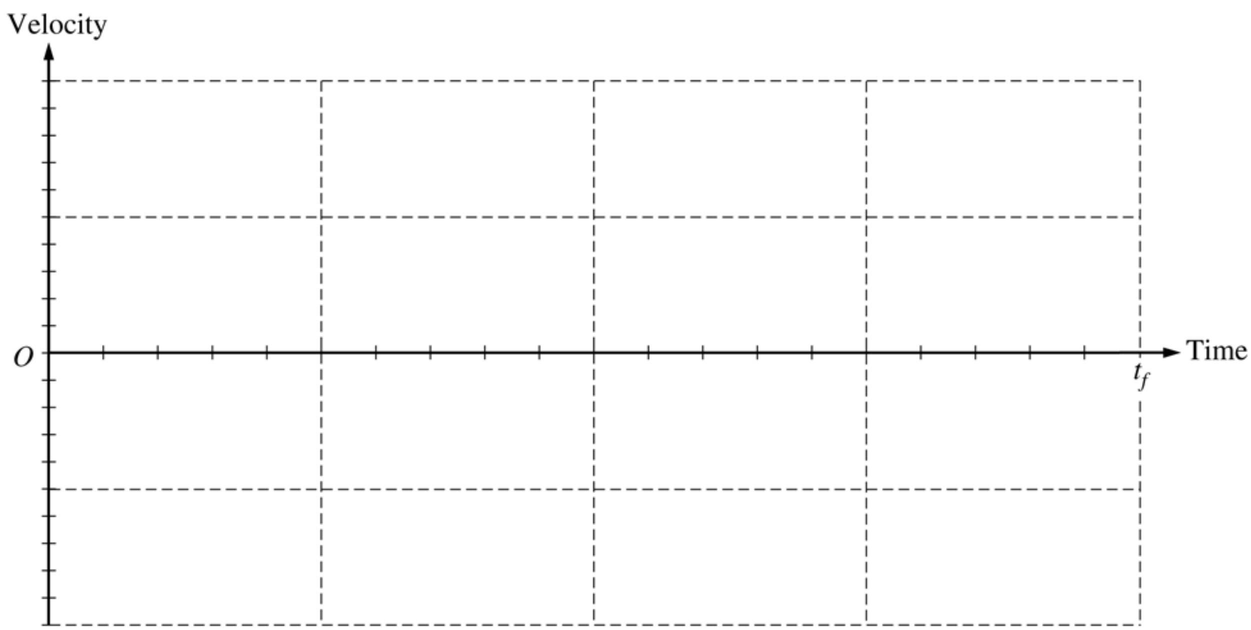
(c) Determine the terminal speed of the ball as it moves downward.

(d) Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

____longer to rise ____longer to fall

Justify your answer.

(e) On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where $t_f$ is the time at which the ball returns to the height from which it was thrown.



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Mech. 2.

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass $M_S$ of Saturn. Assume the orbits of these moons are circular.

| Orbital Period, $T$ (seconds) | Orbital Radius, $R$ (meters) | | |
|---|---|---|---|
| $8.14 \times 10^4$ | $1.85 \times 10^8$ | | |
| $1.18 \times 10^5$ | $2.38 \times 10^8$ | | |
| $1.63 \times 10^5$ | $2.95 \times 10^8$ | | |
| $2.37 \times 10^5$ | $3.77 \times 10^8$ | | |

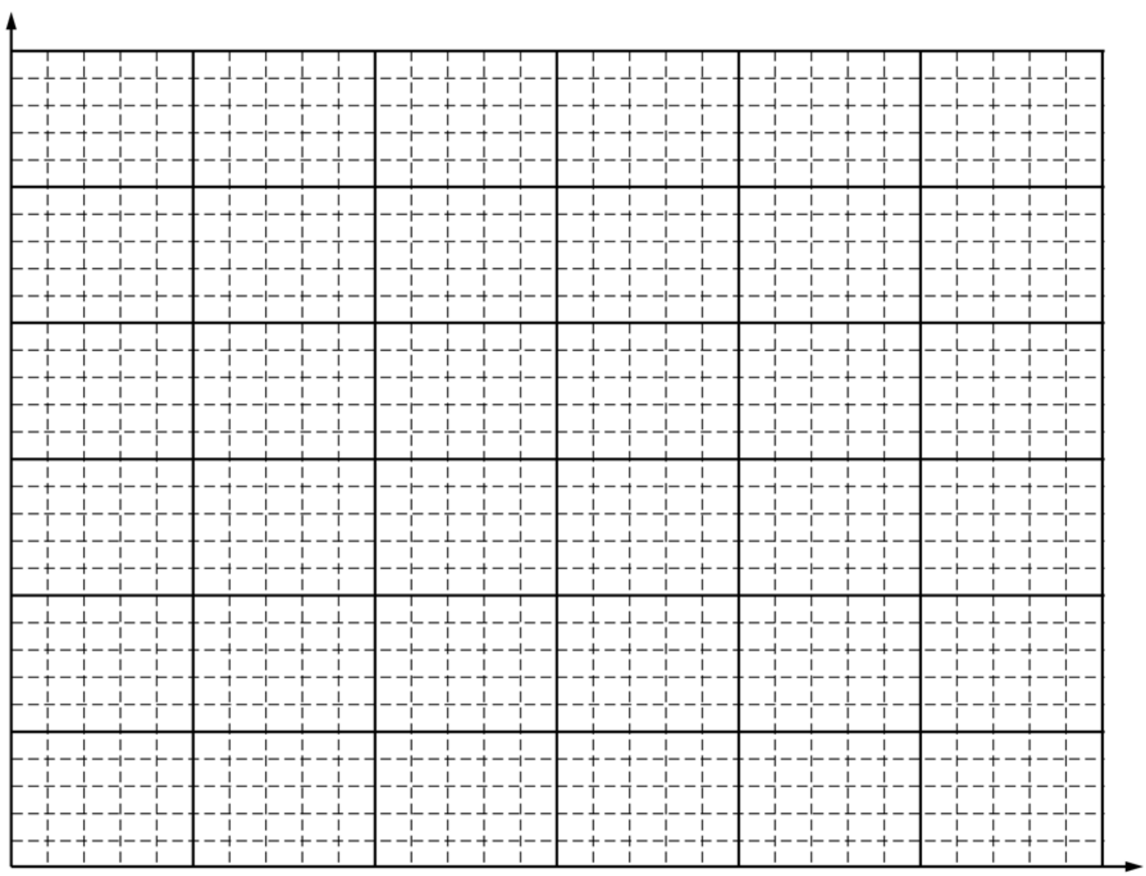
(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period $T$ of a moon as a function of its orbital radius $R$.

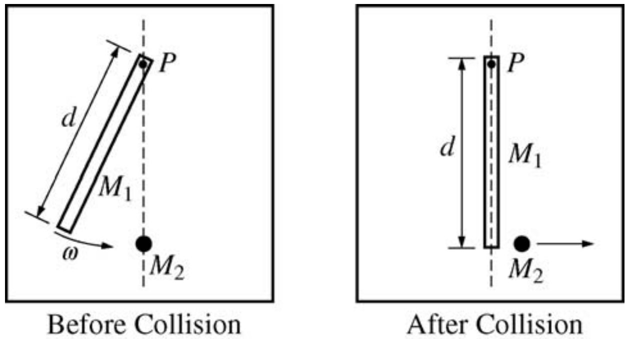
(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.




TOP VIEWS

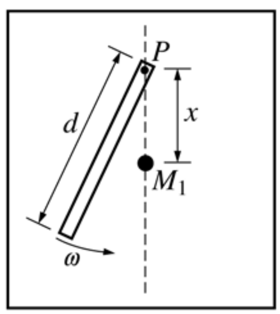
Mech. 3.

A system consists of a ball of mass $M_2$ and a uniform rod of mass $M_1$ and length $d$. The rod is attached to a horizontal frictionless table by a pivot at point $P$ and initially rotates at an angular speed $\omega$, as shown above left. The rotational inertia of the rod about point $P$ is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of $M_1$, $M_2$, $\omega$, $d$, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point $P$ before the collision.

(b) Derive an expression for the speed $v$ of the ball after the collision.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio $M_1/M_2$.



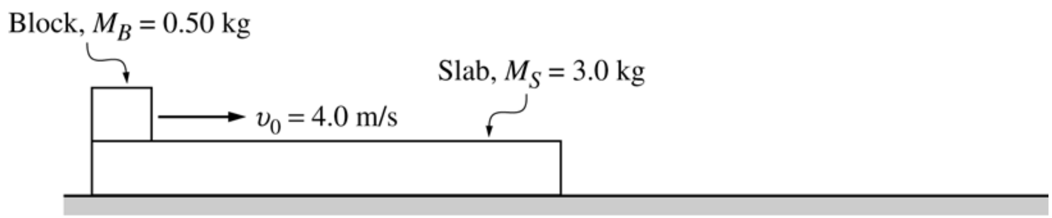

Before Collision

(d) A new ball with the same mass $M_1$ as the rod is now placed a distance $x$ from the pivot, as shown above. Again assuming the collision is elastic, for what value of $x$ will the rod stop moving after hitting the ball?

**END OF SECTION II, MECHANICS**

## PHYSICS C: MECHANICS
## SECTION II
**Time—45 minutes**
**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.




Mech 1.

A small block of mass $M_B = 0.50$ kg is placed on a long slab of mass $M_S = 3.0$ kg as shown above. Initially, the slab is at rest and the block has a speed $v_0$ of 4.0 m/s to the right. The coefficient of kinetic friction between the block and the slab is 0.20, and there is no friction between the slab and the horizontal surface on which it moves.

(a) On the dots below that represent the block and the slab, draw and label vectors to represent the forces acting on each as the block slides on the slab.


At some moment later, before the block reaches the right end of the slab, both the block and the slab attain identical speeds $v_f$.

(b) Calculate $v_f$.

(c) Calculate the distance the slab has traveled at the moment it reaches $v_f$.

(d) Calculate the work done by friction on the slab from the beginning of its motion until it reaches $v_f$.



GO ON TO THE NEXT PAGE.

Mech 2.

A nonlinear spring is compressed various distances $x$, and the force $F$ required to compress it is measured for each distance. The data are shown in the table below.

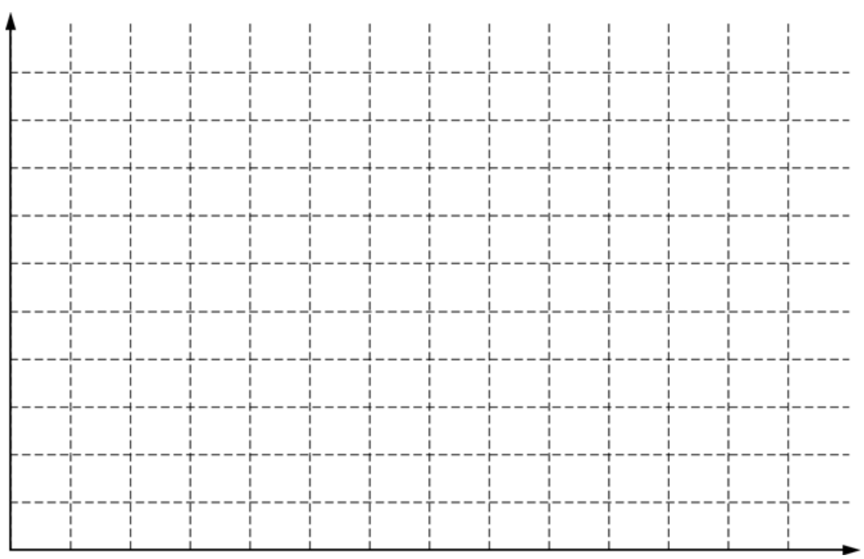
| $x$ (m) | $F$ (N) | |
|---|---|---|
| 0.05 | 4 | |
| 0.10 | 17 | |
| 0.15 | 38 | |
| 0.20 | 68 | |
| 0.25 | 106 | |

Assume that the magnitude of the force applied by the spring is of the form $F(x) = Ax^2$.

(a) Which quantities should be graphed in order to yield a straight line whose slope could be used to calculate a numerical value for $A$ ?

(b) Calculate values for any of the quantities identified in (a) that are not given in the data, and record these values in the table above. Label the top of the column, including units.

(c) On the axes below, plot the quantities you indicated in (a) . Label the axes with the variables and appropriate numbers to indicate the scale.

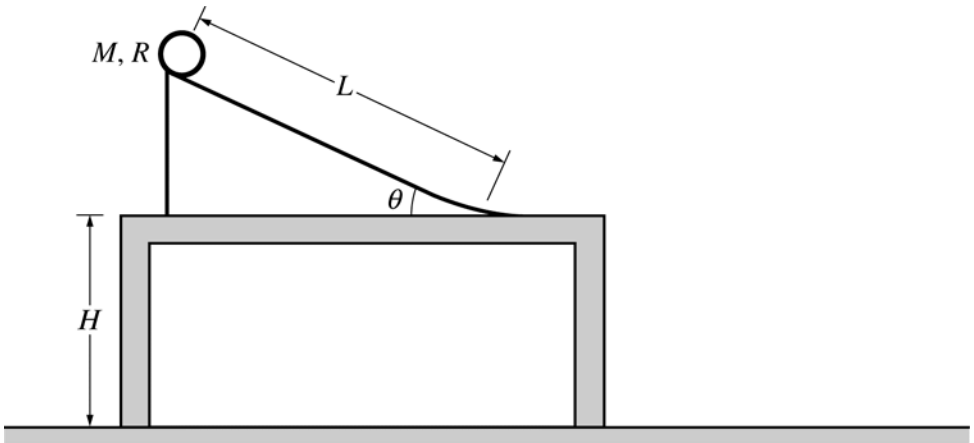


(d) Using your graph, calculate $A$.

The spring is then placed horizontally on the floor. One end of the spring is fixed to a wall. A cart of mass 0.50 kg moves on the floor with negligible friction and collides head-on with the free end of the spring, compressing it a maximum distance of 0.10 m.

(e) Calculate the work done by the cart in compressing the spring 0.10 m from its equilibrium length.

(f) Calculate the speed of the cart just before it strikes the spring.




Mech 3.

A thin hoop of mass $M$, radius $R$, and rotational inertia $MR^2$ is released from rest from the top of the ramp of length $L$ above. The ramp makes an angle $\theta$ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height $H$ above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.

(b) Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

(c) Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.

(d) Suppose that the hoop is now replaced by a disk having the same mass $M$ and radius $R$. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part (c) for the hoop?

____ Less than ____ The same as ____ Greater than

Briefly justify your response.

**END OF EXAM**

## PHYSICS C: MECHANICS
## SECTION II
## Time—45 minutes
## 3 Questions

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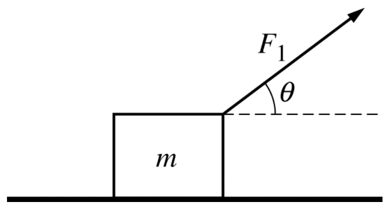
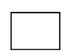



Mech. 1.

A block of mass $m$ is pulled along a rough horizontal surface by a constant applied force of magnitude $F_1$ that acts at an angle $\theta$ to the horizontal, as indicated above. The acceleration of the block is $a_1$. Express all algebraic answers in terms of $m$, $F_1$, $\theta$, $a_1$, and fundamental constants.

(a) On the figure below, draw and label a free-body diagram showing all the forces on the block.



(b) Derive an expression for the normal force exerted by the surface on the block.

(c) Derive an expression for the coefficient of kinetic friction $\mu$ between the block and the surface.

(d) On the axes below, sketch graphs of the speed $v$ and displacement $x$ of the block as functions of time $t$ if the block started from rest at $x = 0$ and $t = 0$.



(e) If the applied force is large enough, the block will lose contact with the surface. Derive an expression for the magnitude of the greatest acceleration $a_{max}$ that the block can have and still maintain contact with the ground.

Mech. 2.

In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of $1.18 \times 10^2$ minutes $= 7.08 \times 10^3$ s and orbital speed of $3.40 \times 10^3$ m/s. The mass of the GS is 930 kg and the radius of Mars is $3.43 \times 10^6$ m.

(a) Calculate the radius of the GS orbit.

(b) Calculate the mass of Mars.

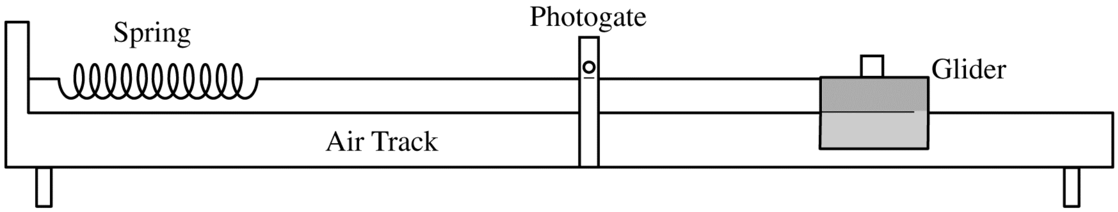
(c) Calculate the total mechanical energy of the GS in this orbit.

(d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_______Greater than _________Less than

Justify your answer.

(e) In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at $3.71 \times 10^5$ m above the surface and its furthest distance at $4.36 \times 10^5$ m above the surface. If the speed of the GS at closest approach is $3.40 \times 10^3$ m/s, calculate the speed at the furthest point of the orbit.



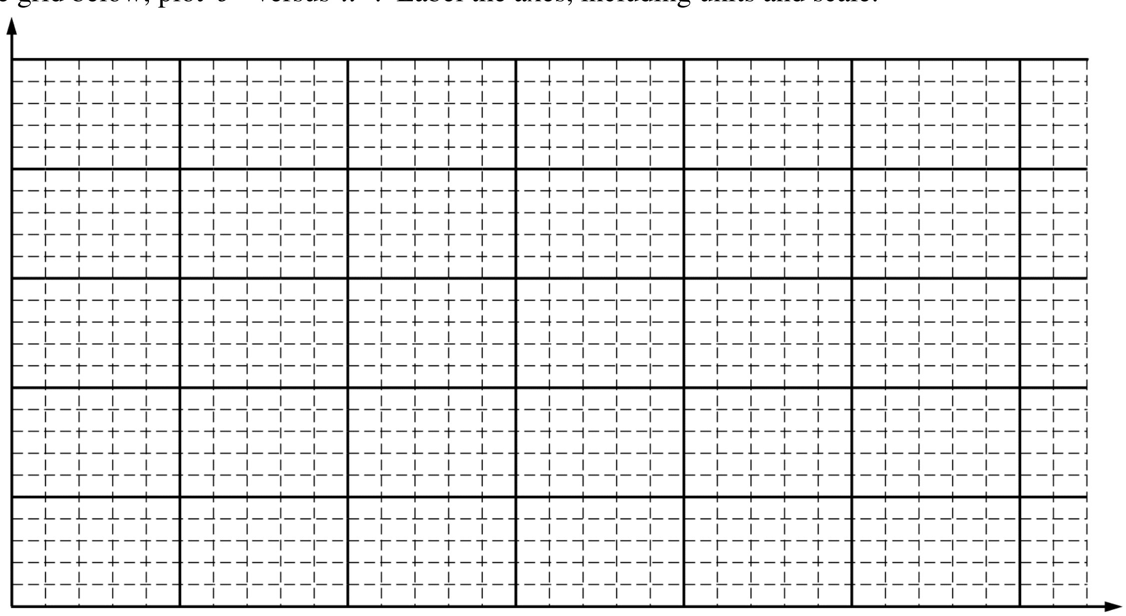

Mech. 3.

The apparatus above is used to study conservation of mechanical energy. A spring of force constant 40 N/m is held horizontal over a horizontal air track, with one end attached to the air track. A light string is attached to the other end of the spring and connects it to a glider of mass $m$ . The glider is pulled to stretch the spring an amount $x$ from equilibrium and then released. Before reaching the photogate, the glider attains its maximum speed and the string becomes slack. The photogate measures the time $t$ that it takes the small block on top of the glider to pass through. Information about the distance $x$ and the speed $v$ of the glider as it passes through the photogate are given below.

| Trial # | Extension of the Spring $x$ (m) | Speed of Glider $v$ (m/s) | Extension Squared $x^2$ $(m^2)$ | Speed Squared $v^2$ $(m^2/s^2)$ |
|---|---|---|---|---|
| 1 | $0.30 \times 10^{-1}$ | 0.47 | $0.09 \times 10^{-2}$ | 0.22 |
| 2 | $0.60 \times 10^{-1}$ | 0.87 | $0.36 \times 10^{-2}$ | 0.76 |
| 3 | $0.90 \times 10^{-1}$ | 1.3 | $0.81 \times 10^{-2}$ | 1.7 |
| 4 | $1.2 \times 10^{-1}$ | 1.6 | $1.4 \times 10^{-2}$ | 2.6 |
| 5 | $1.5 \times 10^{-1}$ | 2.2 | $2.3 \times 10^{-2}$ | 4.8 |

(a) Assuming no energy is lost, write the equation for conservation of mechanical energy that would apply to this situation.

(b) On the grid below, plot $v^2$ versus $x^2$. Label the axes, including units and scale.

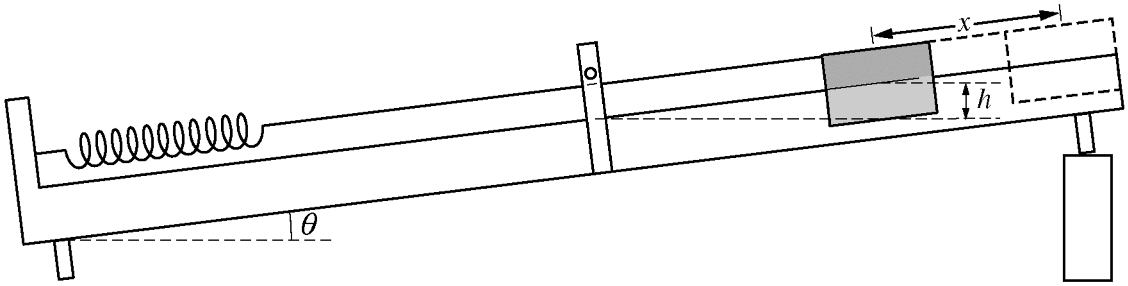


(c)

i. Draw a best-fit straight line through the data.

ii. Use the best-fit line to obtain the mass $m$ of the glider.

(d) The track is now tilted at an angle $\theta$ as shown below. When the spring is unstretched, the center of the glider is a height $h$ above the photogate. The experiment is repeated with a variety of values of $x$.




i. Assuming no energy is lost, write the new equation for conservation of mechanical energy that would apply to this situation.

ii. Will the graph of $v^2$ versus $x^2$ for this new experiment be a straight line?

_____ Yes _____ No

Justify your answer.

**END OF EXAM**

## PHYSICS C: MECHANICS
## SECTION II
**Time—45 minutes**
**3 Questions**

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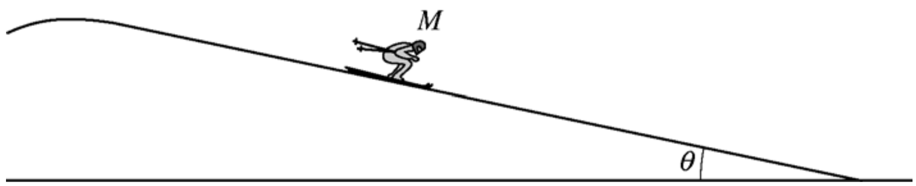
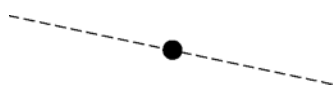



Mech. 1.

A skier of mass $M$ is skiing down a frictionless hill that makes an angle $\theta$ with the horizontal, as shown in the diagram. The skier starts from rest at time $t = 0$ and is subject to a velocity-dependent drag force due to air resistance of the form $F = -bv$, where $v$ is the velocity of the skier and $b$ is a positive constant. Express all algebraic answers in terms of $M$, $b$, $\theta$, and fundamental constants.

(a) On the dot below that represents the skier, draw a free-body diagram indicating and labeling all of the forces that act on the skier while the skier descends the hill.

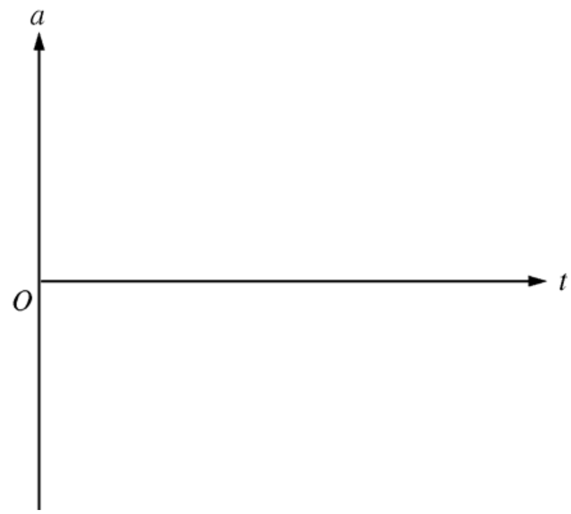


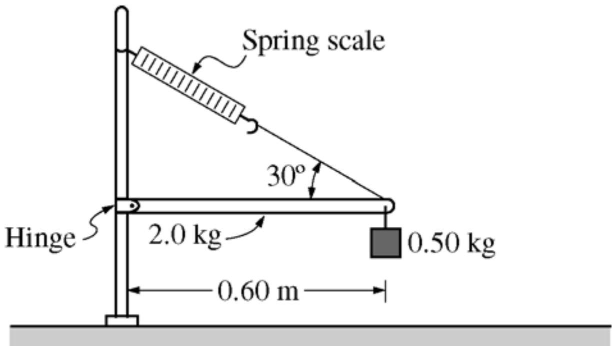
(b) Write a differential equation that can be used to solve for the velocity of the skier as a function of time.

(c) Determine an expression for the terminal velocity $v_T$ of the skier.

(d) Solve the differential equation in part (b) to determine the velocity of the skier as a function of time, showing all your steps.

(e) On the axes below, sketch a graph of the acceleration $a$ of the skier as a function of time $t$, and indicate the initial value of $a$. Take downhill as positive.

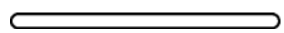





Mech. 2.

The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

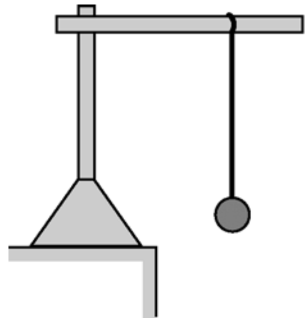
(a) On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.



(b) Calculate the reading on the spring scale.

(c) The rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where $M$ is the mass of the rod and $L$ is its length. Calculate the rotational inertia of the rod-block system about the hinge.

(d) If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

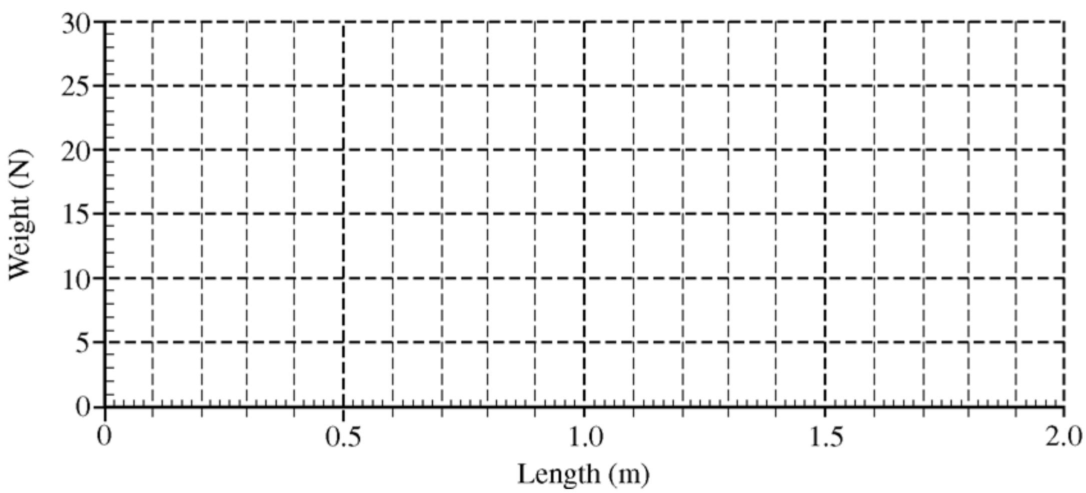


Mech. 3.

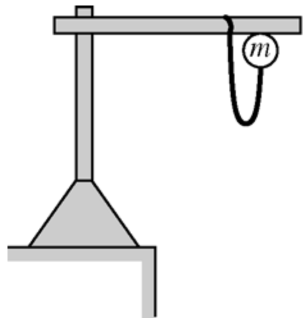
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

| Weight (N) | 0 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| Length (m) | 0.60 | 0.97 | 1.24 | 1.37 | 1.64 |

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.




(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant $k$ of the cord.




The student now attaches an object of unknown mass $m$ to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as represented above. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass $m$ of the object.

(d) i. Calculate how far down the object has fallen at the moment it attains its maximum speed.

ii. Explain why this is the point at which the object has its maximum speed.

iii. Calculate the maximum speed of the object.

**END OF EXAM**

CollegeBoard
Advanced Placement Program

# AP® Physics C: Mechanics
# 2009 Free-Response Questions

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 [27]

**PHYSICS C: MECHANICS**
**SECTION II**
**Time—45 minutes**
**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

Mech. 1.

A 3.0 kg object is moving along the $x$-axis in a region where its potential energy as a function of $x$ is given as $U(x) = 4.0x^2$, where $U$ is in joules and $x$ is in meters. When the object passes the point $x = -0.50$ m, its velocity is +2.0 m/s. All forces acting on the object are conservative.

(a) Calculate the total mechanical energy of the object.

(b) Calculate the $x$-coordinate of any points at which the object has zero kinetic energy.

(c) Calculate the magnitude of the momentum of the object at $x = 0.60$ m.

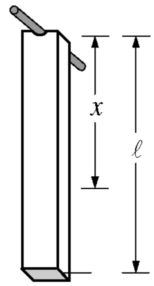
(d) Calculate the magnitude of the acceleration of the object as it passes $x = 0.60$ m.

(e) On the axes below, sketch graphs of the object's position $x$ versus time $t$ and kinetic energy $K$ versus time $t$. Assume that $x = 0$ at time $t = 0$. The two graphs should cover the same time interval and use the same scale on the horizontal axes.





GO ON TO THE NEXT PAGE.




Mech. 2.

You are given a long, thin, rectangular bar of known mass $M$ and length $\ell$ with a pivot attached to one end. The bar has a nonuniform mass density, and the center of mass is located a known distance $x$ from the end with the pivot. You are to determine the rotational inertia $I_b$ of the bar about the pivot by suspending the bar from the pivot, as shown above, and allowing it to swing. Express all algebraic answers in terms of $I_b$, the given quantities, and fundamental constants.

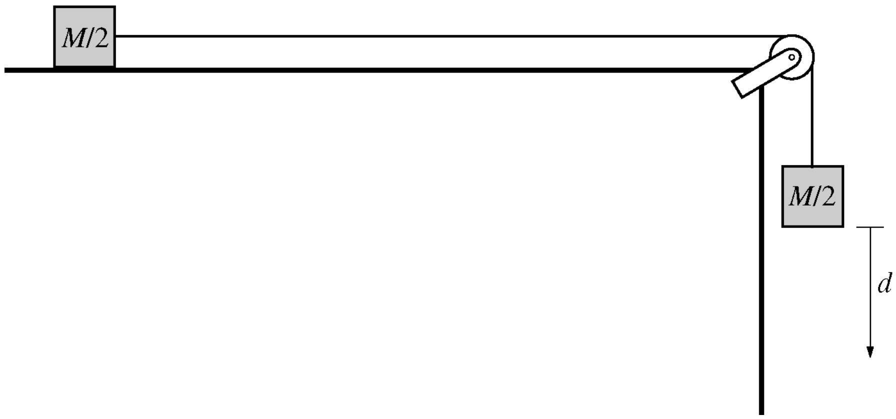
(a)

i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle $\theta$ the bar makes with the vertical.

ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

(b) Describe the experimental procedure you would use to make the additional measurements needed to determine $I_b$. Include how you would use your measurements to obtain $I_b$ and how you would minimize experimental error.

(c) Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.

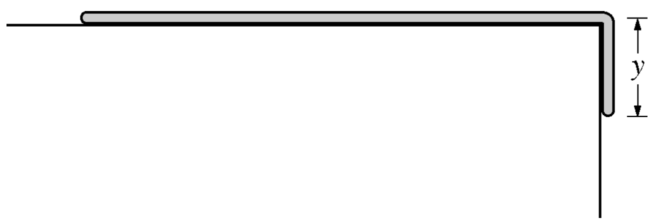



Mech. 3.

A block of mass $M/2$ rests on a frictionless horizontal table, as shown above. It is connected to one end of a string that passes over a massless pulley and has another block of mass $M/2$ hanging from its other end. The apparatus is released from rest.

(a) Derive an expression for the speed $v_h$ of the hanging block as a function of the distance $d$ it descends.

Now the block and pulley system is replaced by a uniform rope of length $L$ and mass $M$, with one end of the rope hanging slightly over the edge of the frictionless table. The rope is released from rest, and at some time later there is a length $y$ of rope hanging over the edge, as shown below. Express your answers to parts (b), (c), and (d) in terms of $y$, $L$, $M$, and fundamental constants.




(b) Determine an expression for the force of gravity on the hanging part of the rope as a function of $y$.

(c) Derive an expression for the work done by gravity on the rope as a function of $y$, assuming $y$ is initially zero.

(d) Derive an expression for the speed $v_r$ of the rope as a function of $y$.

(e) The hanging block and the right end of the rope are each allowed to fall a distance $L$ (the length of the rope). The string is long enough that the sliding block does not hit the pulley. Indicate whether $v_h$ from part (a) or $v_r$ from part (d) is greater after the block and the end of the rope have traveled this distance.

____ $v_h$ is greater. ____ $v_r$ is greater. ____ The speeds are equal.

Justify your answer.

**END OF EXAM**

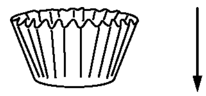
**PHYSICS C: MECHANICS**

**SECTION II**

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech. 1.

Students are to conduct an experiment to investigate the relationship between the terminal speed of a stack of falling paper coffee filters and its mass. Their procedure involves stacking a number of coffee filters, like the one shown in the figure above, and dropping the stack from rest. The students change the number of filters in the stack to vary the mass $m$ while keeping the shape of the stack the same. As a stack of coffee filters falls, there is an air resistance (drag) force acting on the filters.

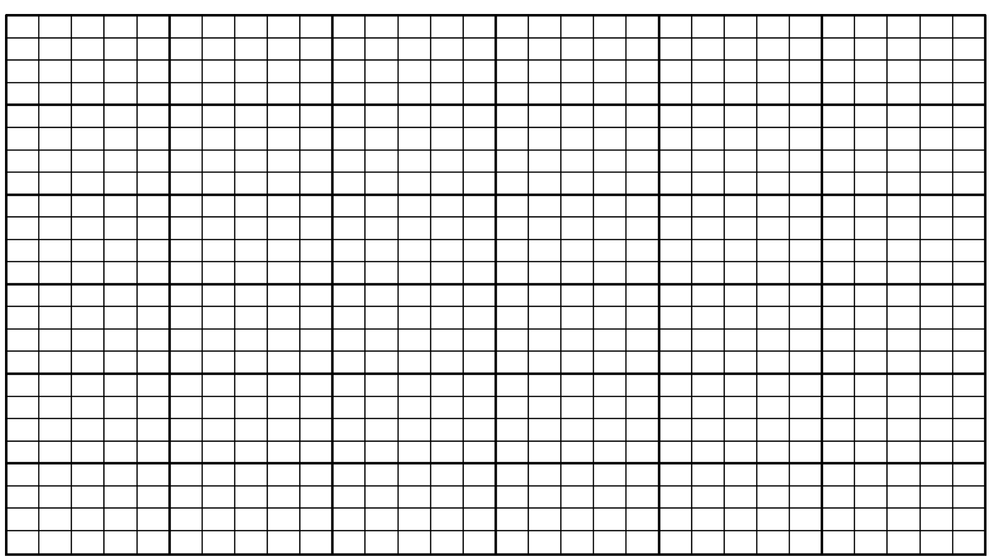
(a) The students suspect that the drag force $F_D$ is proportional to the square of the speed $v$ : $F_D = Cv^2$ , where $C$ is a constant. Using this relationship, derive an expression relating the terminal speed $v_T$ to the mass $m$.

The students conduct the experiment and obtain the following data.

| | | | | | |
|---|---|---|---|---|---|
| Mass of the stack of filters, $m$ (kg) | $1.12 \times 10^{-3}$ | $2.04 \times 10^{-3}$ | $2.96 \times 10^{-3}$ | $4.18 \times 10^{-3}$ | $5.10 \times 10^{-3}$ |
| Terminal speed, $v_T$ ( m/s ) | 0.51 | 0.62 | 0.82 | 0.92 | 1.06 |
| | | | | | |

(b)

(i) Assuming the functional relationship for the drag force above, use the grid below to plot a linear graph as a function of $m$ to verify the relationship. Use the empty boxes in the data table, as appropriate, to record any calculated values you are graphing. Label the vertical axis as appropriate, and place numbers on both axes.



$m$ (kg)

(ii) Use your graph to calculate $C$.

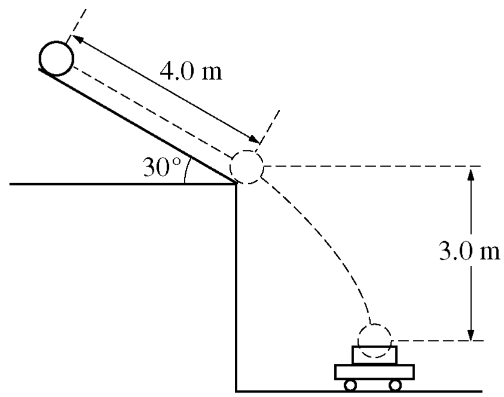
A particular stack of filters with mass $m$ is dropped from rest and reaches a speed very close to terminal speed by the time it has fallen a vertical distance $Y$.

(c)

(i) Sketch an approximate graph of speed versus time from the time the filters are released up to the time $t = T$ that the filters have fallen the distance $Y$. Indicate time $t = T$ and terminal speed $v = v_T$ on the graph.


(ii) Suppose you had a graph like the one sketched in (c)(i) that had a numerical scale on each axis. Describe how you could use the graph to approximate the distance $Y$.

(d) Determine an expression for the approximate amount of mechanical energy dissipated, $\Delta E$, due to air resistance during the time the stack falls a distance $y$, where $y > Y$. Express your answer in terms of $y$, $m$, $v_T$, and fundamental constants.

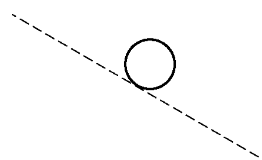



Note: Figure not drawn to scale.

Mech. 2.

A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at $30°$, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass $M$ and radius $R$ about its center of mass is $\frac{2}{5}MR^2$.

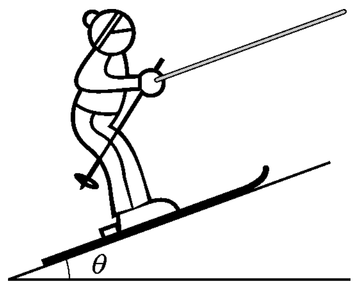
(a) On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.



(b) Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

(d) A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.




Mech. 3.

A skier of mass $m$ will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time $t$ can be modeled by the equations

$$a = a_{max} \sin\frac{\pi t}{T} \qquad (0 < t < T)$$
$$= 0 \qquad (t \geq T),$$

where $a_{max}$ and $T$ are constants. The hill is inclined at an angle $\theta$ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

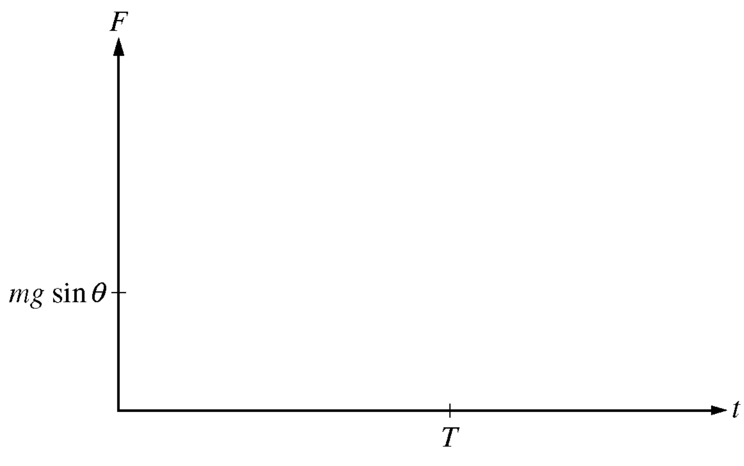
(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

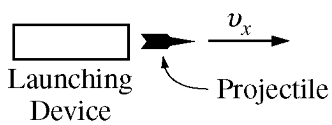
(e) Suppose that the magnitude of the acceleration is instead modeled as $a = a_{max}e^{-\pi t/2T}$ for all $t > 0$, where $a_{max}$ and $T$ are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model $F_1$ and the new model $F_2$.




**END OF EXAM**

## PHYSICS C: MECHANICS
## SECTION II
## Time—45 minutes
## 3 Questions

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.




Mech. 1.

A projectile is fired horizontally from a launching device, exiting with a speed $v_x$. While the projectile is in the launching device, the impulse imparted to it is $J_p$, and the average force on it is $F_{avg}$. Assume the force becomes zero just as the projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of $v_x$, $J_p$, $F_{avg}$, and fundamental constants, as appropriate.

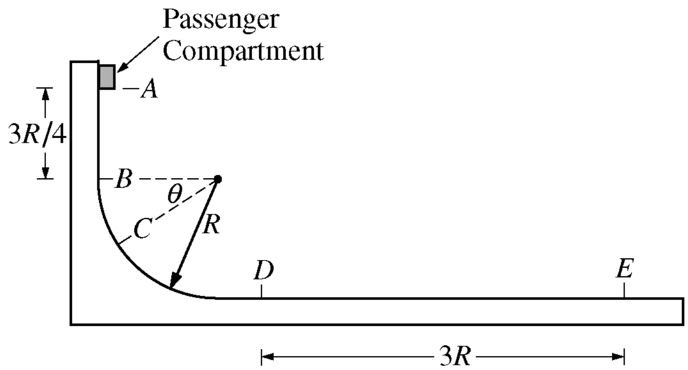
(a) Determine an expression for the time required for the projectile to travel the length of the launching device.
(b) Determine an expression for the mass of the projectile.

The projectile is fired horizontally into a block of wood that is clamped to a tabletop so that it cannot move. The projectile travels a distance $d$ into the block before it stops. Express all algebraic answers to the following in terms of $d$ and the given quantities previously indicated, as appropriate.

(c) Derive an expression for the work done in stopping the projectile.
(d) Derive an expression for the average force $F_b$ exerted on the projectile as it comes to rest in the block.

Now a new projectile and block are used, identical to the first ones, but the block is not clamped to the table. The projectile is again fired into the block of wood and travels a new distance $d_n$ into the block while the block slides across the table a short distance $D$. Assume the following: the projectile enters the block with speed $v_x$, the average force $F_b$ between the projectile and the block has the same value as determined in part (d), the average force of friction between the table and the block is $f_T$, and the collision is instantaneous so the frictional force is negligible during the collision.

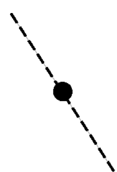
(e) Derive an expression for $d_n$ in terms of $d$, $D$, $f_T$, and $F_b$, as appropriate.
(f) Derive an expression for $d_n$ in terms of $d$, the mass $m$ of the projectile, and the mass $M$ of the block.




Mech. 2.

An amusement park ride features a passenger compartment of mass $M$ that is released from rest at point $A$, as shown in the figure above, and moves along a track to point $E$. The compartment is in free fall between points $A$ and $B$, which are a distance of $3R/4$ apart, then moves along the circular arc of radius $R$ between points $B$ and $D$. Assume the track is frictionless from point $A$ to point $D$ and the dimensions of the passenger compartment are negligible compared to $R$.

(a) On the dot below that represents the passenger compartment, draw and label the forces (not components) that act on the passenger compartment when it is at point $C$, which is at an angle $\theta$ from point $B$.



(b) In terms of $\theta$ and the magnitudes of the forces drawn in part (a), determine an expression for the magnitude of the centripetal force acting on the compartment at point $C$. If you need to draw anything besides what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Derive an expression for the speed $v_D$ of the passenger compartment as it reaches point $D$ in terms of $M$, $R$, and fundamental constants, as appropriate.

A force acts on the compartment between points $D$ and $E$ and brings it to rest at point $E$.

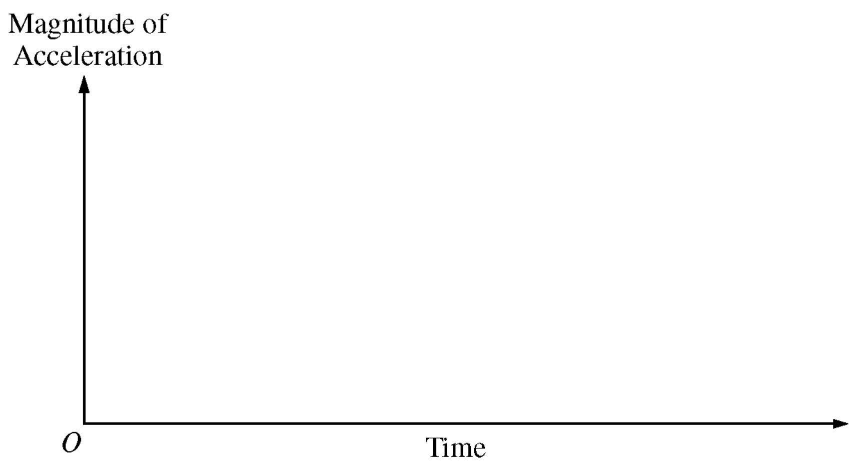
(d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction $\mu$ between the compartment and the track.

(e) Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force $-k\mathbf{v}$, where $k$ is a constant and $\mathbf{v}$ is the velocity of the compartment. Express all algebraic answers to the following in terms of $M$, $R$, $k$, $\upsilon_D$, and fundamental constants, as appropriate.

i. Write, but do NOT solve, the differential equation for $\upsilon(t)$.

ii. Solve the differential equation you wrote in part i.

iii. On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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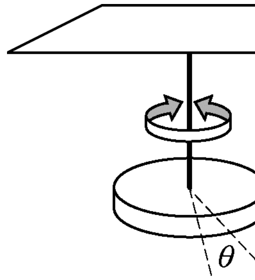



Mech. 3.

The torsion pendulum shown above consists of a disk of rotational inertia $I$ suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle $\theta$, the twisted rod exerts a restoring torque $\tau$ that is proportional to the angular displacement: $\tau = -\beta\theta$, where $\beta$ is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.

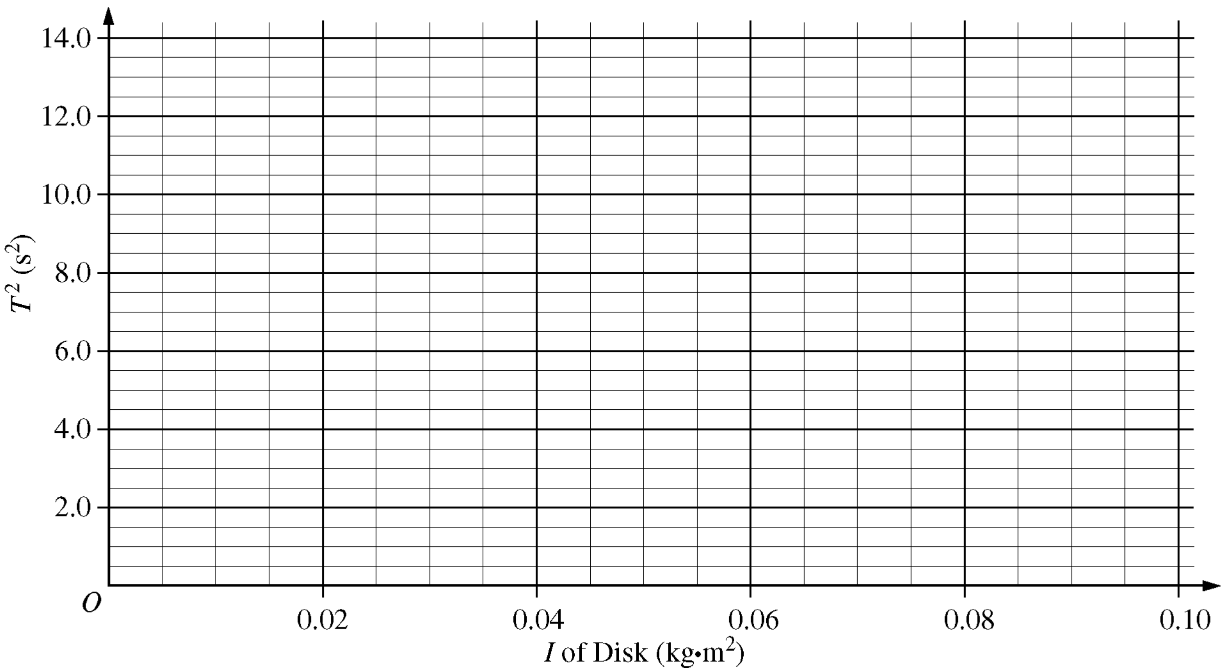
(a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement $\theta$ of the torsion pendulum as a function of time $t$.

(b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.

To determine the torsion constant $\beta$ of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

| Rotational Inertia $I$ of Disk $(kg{\bullet}m^2)$ | Average Time for Ten Oscillations (s) | Period $T$ (s) | $T^2$ $(s^2)$ |
|---|---|---|---|
| 0.025 | 22.4 | 2.24 | 5.0 |
| 0.036 | 26.8 | 2.68 | 7.2 |
| 0.049 | 29.5 | 2.95 | 8.7 |
| 0.064 | 33.3 | 3.33 | 11.1 |
| 0.081 | 35.9 | 3.59 | 12.9 |

(c) On the graph below, plot the data points. Draw a straight line that best represents the data.




(d) Determine the equation for your line.

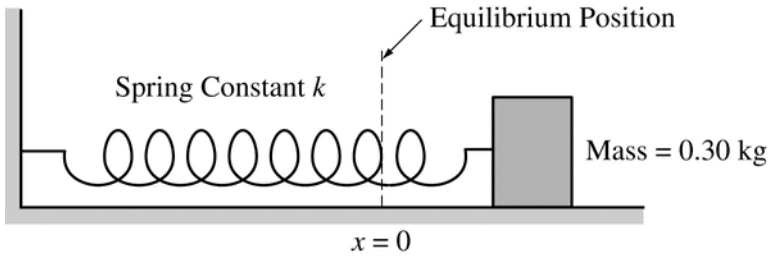
(e) Calculate the torsion constant $\beta$ of the rod from your line.

(f) What is the physical significance of the intercept of your line with the vertical axis?

**END OF EXAM**

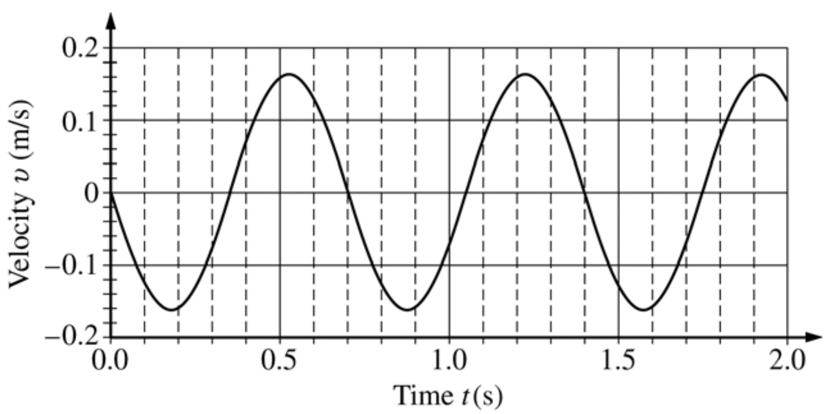
**PHYSICS C: MECHANICS**
**SECTION II**
**Time—45 minutes**
**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




Mech. 1.

Experiment 1. A block of mass 0.30 kg is placed on a frictionless table and is attached to one end of a horizontal spring of spring constant $k$, as shown above. The other end of the spring is attached to a fixed wall. The block is set into oscillatory motion by stretching the spring and releasing the block from rest at time $t = 0$. A motion detector is used to record the position of the block as it oscillates. The resulting graph of velocity $v$ versus time $t$ is shown below. The positive direction for all quantities is to the right.

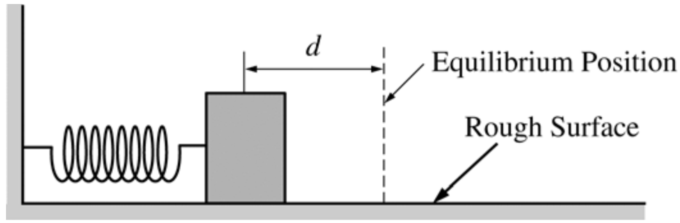



(a) Determine the equation for $v(t)$, including numerical values for all constants.

(b) Given that the equilibrium position is at $x = 0$, determine the equation for $x(t)$, including numerical values for all constants.

(c) Calculate the value of $k$.

Experiment 2. The block and spring arrangement is now placed on a rough surface, as shown below. The block is displaced so that the spring is compressed a distance $d$ and released from rest.

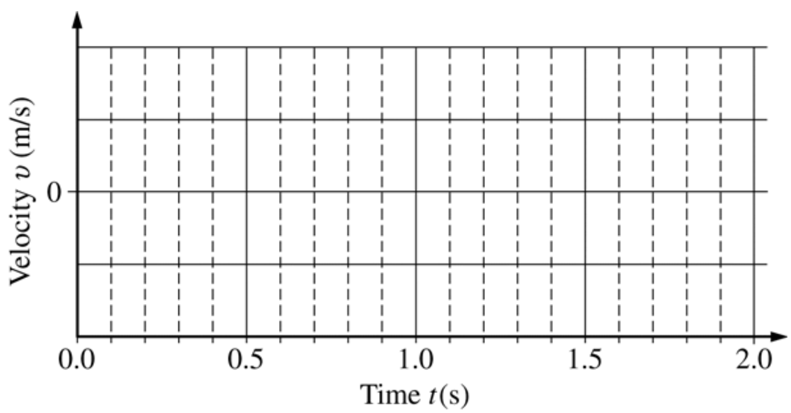



(d) On the dots below that represent the block, draw and label the forces (not components) that act on the block when the spring is compressed a distance $x = d/2$ and the block is moving in the direction indicated below each dot.

Toward the equilibrium position

Away from the equilibrium position

(e) Draw a sketch of $v$ versus $t$ in this case. Assume that there is a negligible change in the period and that the positive direction is still to the right.



Mech. 2.

You are to perform an experiment investigating the conservation of mechanical energy involving a transformation from initial gravitational potential energy to translational kinetic energy.

(a) You are given the equipment listed below, all the supports required to hold the equipment, and a lab table. On the list below, indicate each piece of equipment you would use by checking the line next to each item.

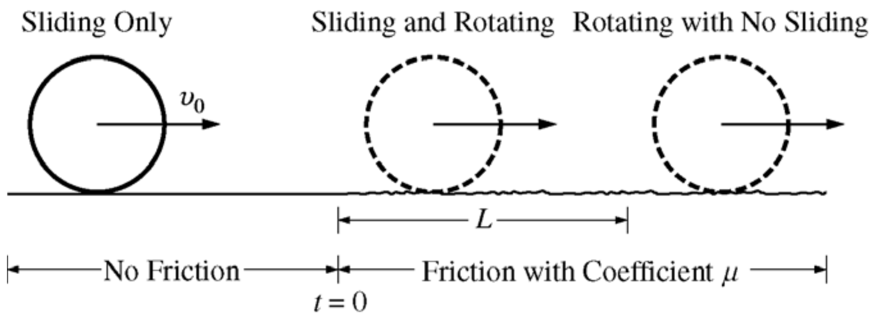
| | | |
|---|---|---|
| ____ Track | ____ Meterstick | ____ Set of objects of different masses |
| ____ Cart | ____ Electronic balance | ____ Lightweight low-friction pulley |
| ____ String | ____ Stopwatch | |

(b) Outline a procedure for performing the experiment. Include a diagram of your experimental setup. Label the equipment in your diagram. Also include a description of the measurements you would make and a symbol for each measurement.

(c) Give a detailed account of the calculations of gravitational potential energy and translational kinetic energy both before and after the transformation, in terms of the quantities measured in part (b).

(d) After your first trial, your calculations show that the energy <u>increased</u> during the experiment. Assuming you made no mathematical errors, give a reasonable explanation for this result.

(e) On all other trials, your calculations show that the energy <u>decreased</u> during the experiment. Assuming you made no mathematical errors, give a reasonable physical explanation for the fact that the average energy you determined decreased. Include references to conservative and nonconservative forces, as appropriate.




Mech. 3.

A ring of mass $M$, radius $R$, and rotational inertia $MR^2$ is initially sliding on a frictionless surface at constant velocity $v_0$ to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction $\mu$ and begins sliding and rotating. After traveling a distance $L$, the ring begins rolling without sliding. Express all answers to the following in terms of $M$, $R$, $v_0$, $\mu$, and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity $v$ of the ring as a function of time $t$

ii. The angular velocity $\omega$ of the ring as a function of time $t$

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity $v$ of the ring as a function of time $t$

ii. The angular velocity $\omega$ of the ring as a function of time $t$

(c) Derive an expression for the time it takes the ring to travel the distance $L$.

(d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance $L$.

(e) Derive an expression for the distance $L$.

**STOP**

**END OF EXAM**

**PHYSICS C: MECHANICS**
**SECTION II**
**Time—45 minutes**
**3 Questions**

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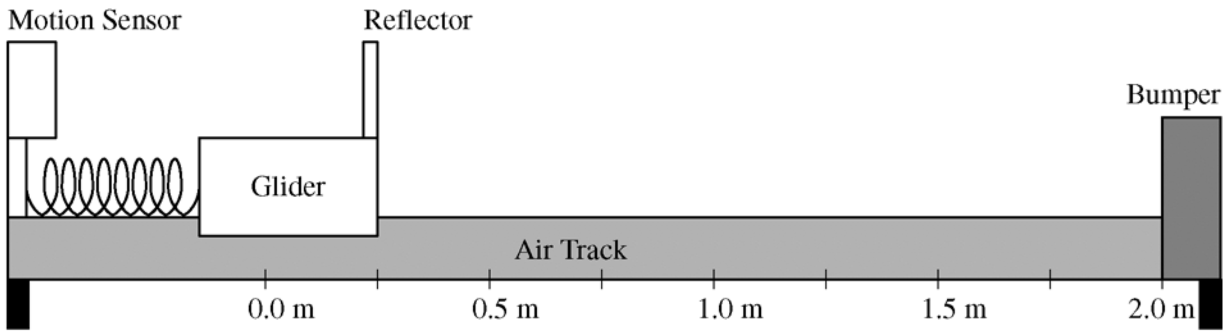



Figure 1

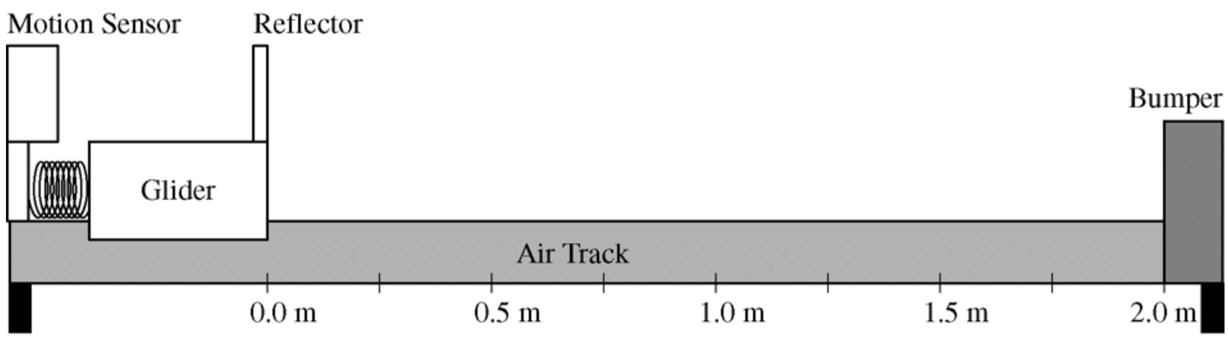


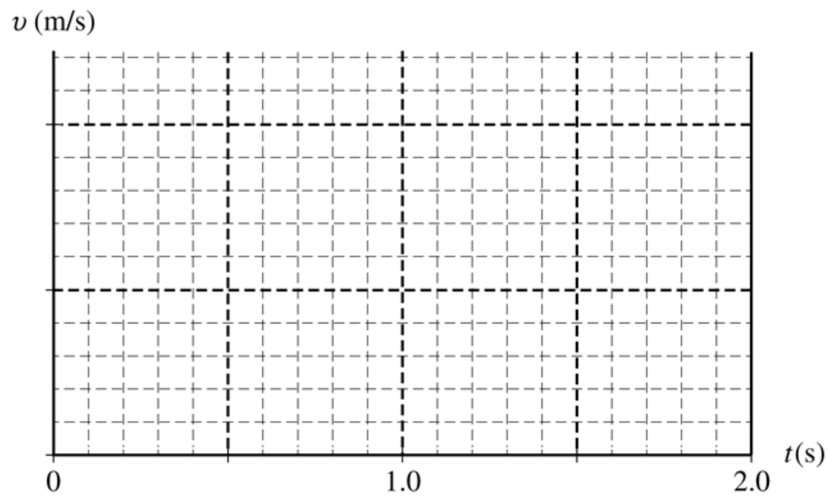

Figure 2

Mech 1.

A student places a 0.40 kg glider on an air track of negligible friction and holds it so that it touches an uncompressed ideal spring, as shown in Figure 1 above. The student then pushes the glider back to compress the spring by 0.25 m, as shown in Figure 2. At time $t = 0$, the student releases the glider, and a motion sensor begins recording the velocity of the reflector at the front of the glider as a function of time. The data points are shown in the table below. At time $t = 0.79$ s, the glider loses contact with the spring.

| Time (s) | 0 | 0.25 | 0.50 | 0.75 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|
| Velocity (m/s) | 0 | 0.25 | 0.43 | 0.48 | 0.50 | 0.49 | 0.51 |

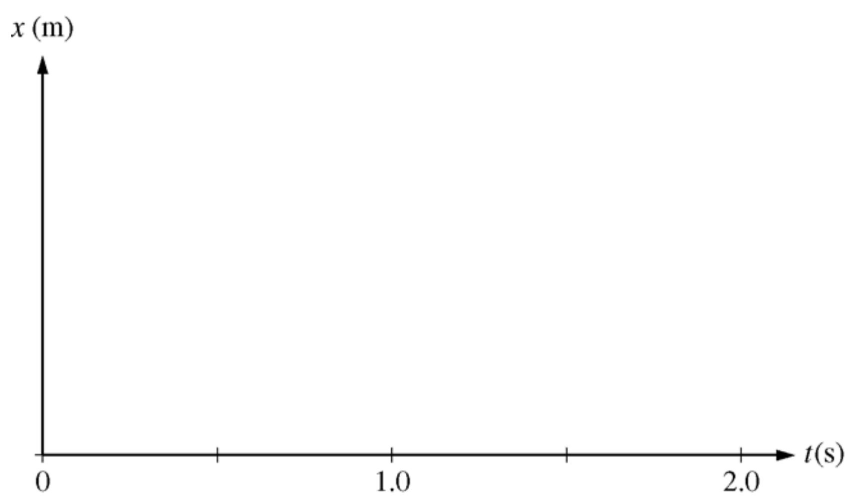
(a) On the axes below, plot the data points for velocity $v$ as a function of time $t$ for the glider, and draw a smooth curve that best fits the data. Be sure to label an appropriate scale on the vertical axis.




(b) The student wishes to use the data to plot position $x$ as a function of time $t$ for the glider.

i. Describe a method the student could use to do this.

ii. On the axes below, sketch the position $x$ as a function of time $t$ for the glider. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



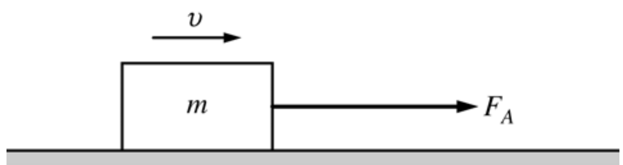

(c) Calculate the time at which the glider makes contact with the bumper at the far right.

(d) Calculate the force constant of the spring.

(e) The experiment is run again, but this time the glider is attached to the spring rather than simply being pushed against it.

i. Determine the amplitude of the resulting periodic motion.

ii. Calculate the period of oscillation of the resulting periodic motion.




Mech 2.

A box of mass $m$ initially at rest is acted upon by a constant applied force of magnitude $F_A$, as shown in the figure above. The friction between the box and the horizontal surface can be assumed to be negligible, but the box is subject to a drag force of magnitude $kv$ where $v$ is the speed of the box and $k$ is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.

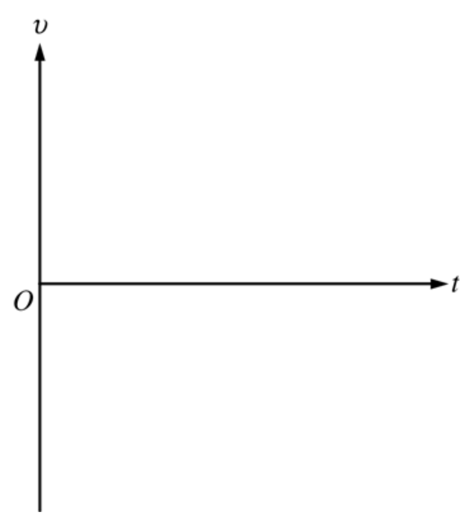
(a) The dot below represents the box. Draw and label the forces (not components) that act on the box.

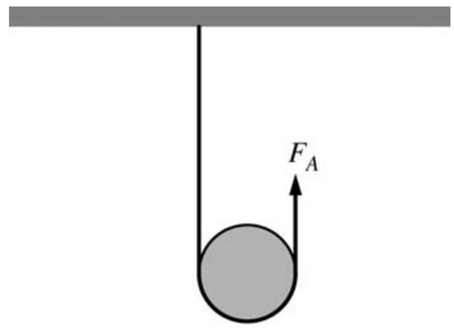
(b) Write, but do not solve, a differential equation that could be used to determine the speed $v$ of the box as a function of time $t$. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Determine the magnitude of the terminal velocity of the box.

(d) Use the differential equation from part (b) to derive the equation for the speed $v$ of the box as a function of time $t$. Assume that $v = 0$ at time $t = 0$.

(e) On the axes below, sketch a graph of the speed $v$ of the box as a function of time $t$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.






Note: Figure not drawn to scale.

Mech 3.

A disk of mass $M = 2.0$ kg and radius $R = 0.10$ m is supported by a rope of negligible mass, as shown above. The rope is attached to the ceiling at one end and passes under the disk. The other end of the rope is pulled upward with a force $F_A$. The rotational inertia of the disk around its center is $MR^2/2$.

(a) Calculate the magnitude of the force $F_A$ necessary to hold the disk at rest.

At time $t = 0$, the force $F_A$ is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.

(b) Calculate the linear acceleration of the disk.

(c) Calculate the angular speed of the disk at $t = 3.0$ s.

(d) Calculate the increase in total mechanical energy of the disk from $t = 0$ to $t = 3.0$ s.

(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.

__ Greater than      __ Less than      __ The same as

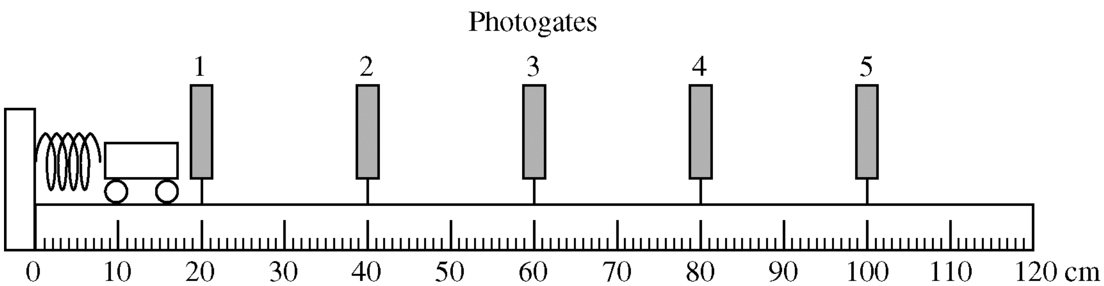
Justify your answer.

**STOP**

**END OF EXAM**

**PHYSICS C: MECHANICS**
**SECTION II**
**Time—45 minutes**
**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where $s$ is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy $U$ as a function of the compression $s$. Express your answer in terms of $A$, $B$, $s$, and fundamental constants, as appropriate.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \text{N/m}^2$ and $B = 150\ \text{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

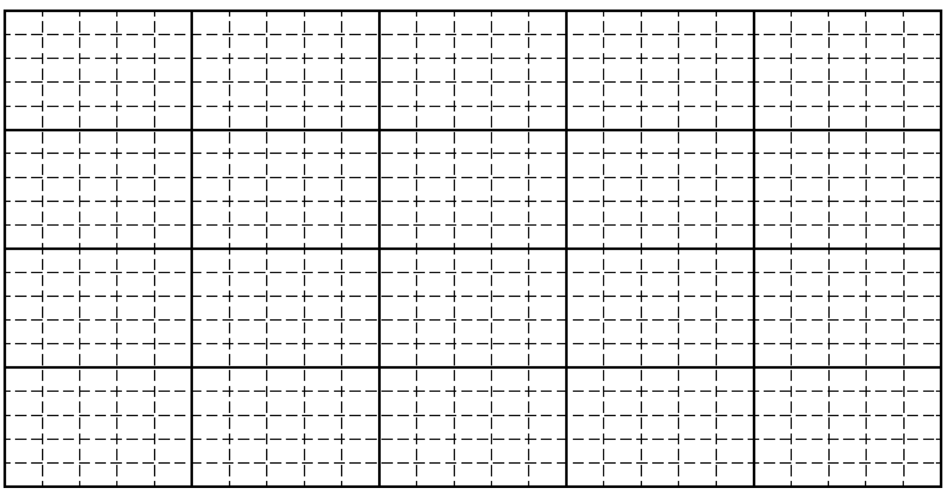
i. The speed of the cart immediately after it loses contact with the spring

ii. The impulse given to the cart by the spring

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of $0.320\ \text{m/s}$ after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \text{m/s}$. The positions are measured with a precision of $\pm 0.005$ m.

| Photogate | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cart speed (m/s) | 0.412 | 0.407 | 0.399 | 0.374 | 0.338 |
| Photogate position (m) | 0.20 | 0.40 | 0.60 | 0.80 | 1.00 |

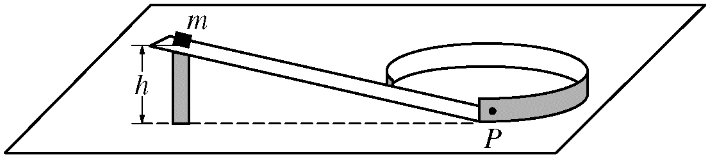
(c) On the axes below, plot the data points for the speed $v$ of the cart as a function of position $x$. Clearly scale and label all axes, as appropriate.



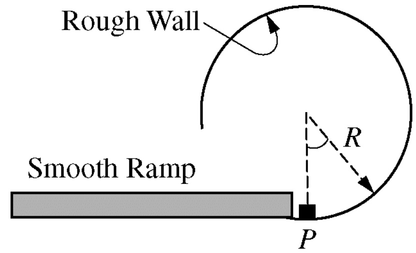
(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.




Side View




Top View

Mech. 2.

A small block of mass $m$ starts from rest at the top of a frictionless ramp, which is at a height $h$ above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point $P$ with a speed $v_0$. After the block reaches point $P$ at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius $R$, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is $\mu$.

(a) Derive an expression for the height of the ramp $h$. Express your answer in terms of $v_0$, $m$, and fundamental constants, as appropriate.

A short time after passing point $P$, the block is in contact with the wall and moves with a speed of $v$.

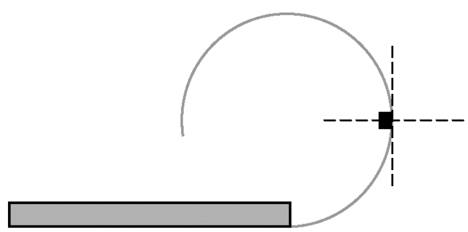
(b)

i. Is the vertical component of the net force on the block upward, downward, or zero?

____ Upward ____ Downward ____ Zero

Justify your answer.

ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Top View

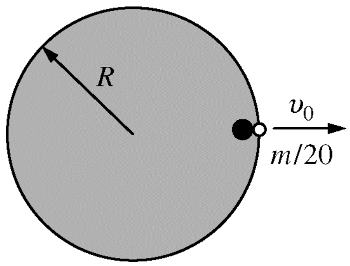
Justify your answer.

Express your answers to the following in terms of $v_0$, $v$, $m$, $R$, $\mu$, and fundamental constants, as appropriate.

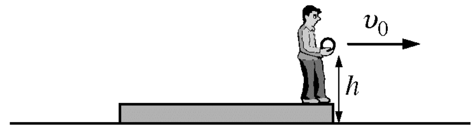
(c) Determine an expression for the magnitude of the normal force $N$ exerted on the block by the circular wall as a function of $v$.

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of $v$.

(e) Derive an expression for $v(t)$, the speed of the block as a function of time $t$ after passing point $P$ on the track.




Top View




Side View

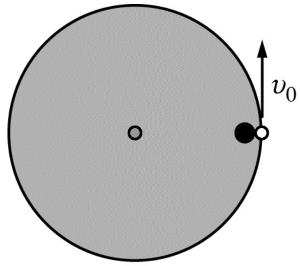
Mech. 3.

A large circular disk of mass $m$ and radius $R$ is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed $v_0$ from a height $h$ above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is $\mu$. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of $m$, $R$, $v_0$, $h$, $\mu$, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

(c) Derive an expression for the time it will take the disk to stop sliding.



Top View

The person now stands on a similar disk of mass $m$ and radius $R$ that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed $v_0$, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

(d) Derive an expression for the angular speed $\omega$ of the disk immediately after the stone is thrown.

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed $v_0$ in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d) ?

____ Greater than ____ Less than ____ Equal to

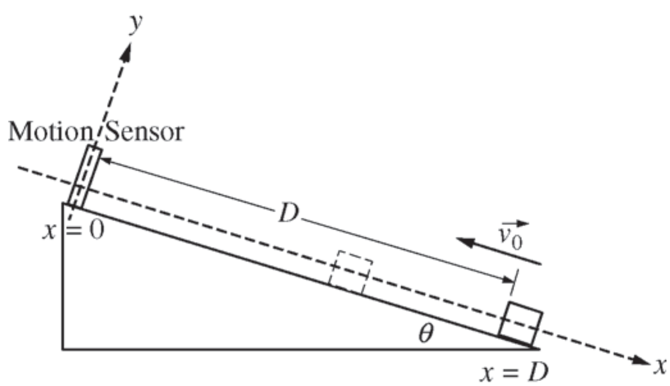
Justify your answer.

**STOP**

**END OF EXAM**

**PHYSICS C: MECHANICS**
**SECTION II**
**Time—45 minutes**
**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




Mech.1.

A block of mass $m$ is projected up from the bottom of an inclined ramp with an initial velocity of magnitude $v_0$. The ramp has negligible friction and makes an angle $\theta$ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance $D$ from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time $t$ after it has been projected up the ramp. Express your answers in terms of $m$, $D$, $v_0$, $t$, $\theta$ and physical constants, as appropriate.

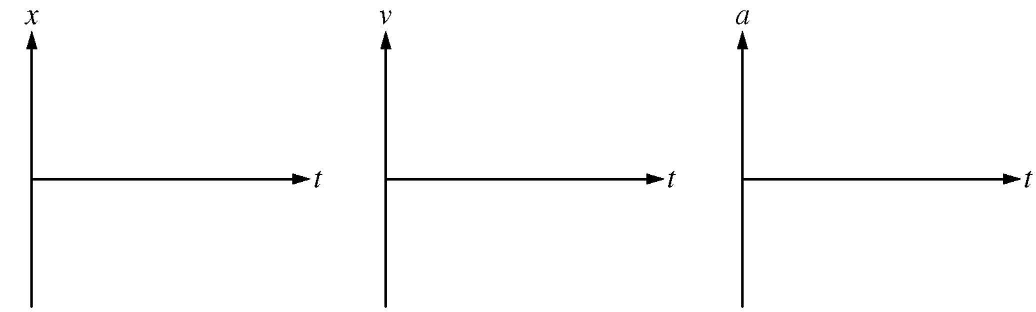
i. Determine the acceleration $a$ of the block.

ii. Determine an expression for the velocity $v$ of the block.

iii. Determine an expression for the position $x$ of the block.

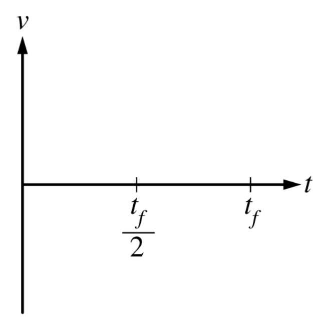
(b) Derive an expression for the position $x_{min}$ of the block when it is closest to the motion sensor. Express your answer in terms of $m$, $D$, $v_0$, $\theta$, and physical constants, as appropriate.

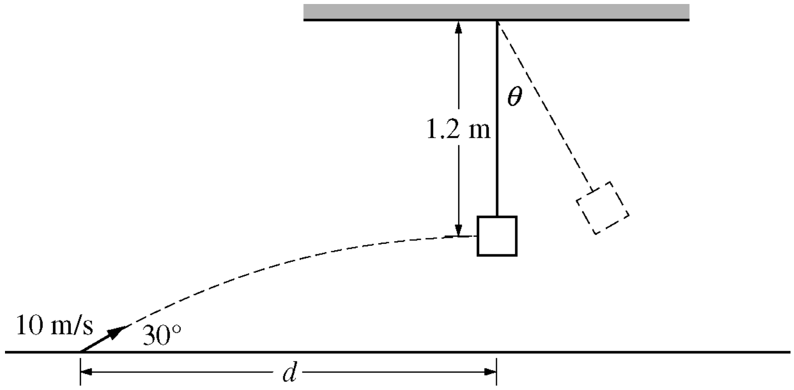
(c) On the axes provided below, sketch graphs of position $x$, velocity $v$, and acceleration $a$ as functions of time $t$ for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.




(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by $\mu_k$. Derive an expression for the distance the block slides before stopping. Express your answer in terms of $m$, $D$, $v_0$, $\theta$, $\mu_k$, and physical constants, as appropriate.

(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed $v_0$ and comes back down the ramp. On the axes provided below, sketch a graph of the velocity $v$ as a function of time $t$ for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time $t_f$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.






Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle $\theta$ with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

(b) Calculate the horizontal distance $d$ between the launching point of the dart and a point on the floor directly below the block.

(c) Calculate the speed of the block just after the dart strikes.

(d) Calculate the angle $\theta$ through which the dart and block on the string will rise before coming momentarily to rest.

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

i. Would the angle $\theta$ that the dart and block swing to increase, decrease, or stay the same?

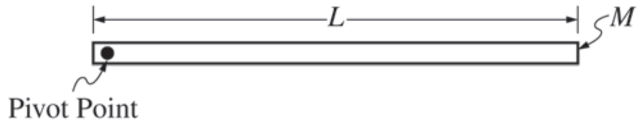
_____ Increase _____ Decrease _____ Stay the same

Justify your answer.

ii. Would the period of oscillation after the collision increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

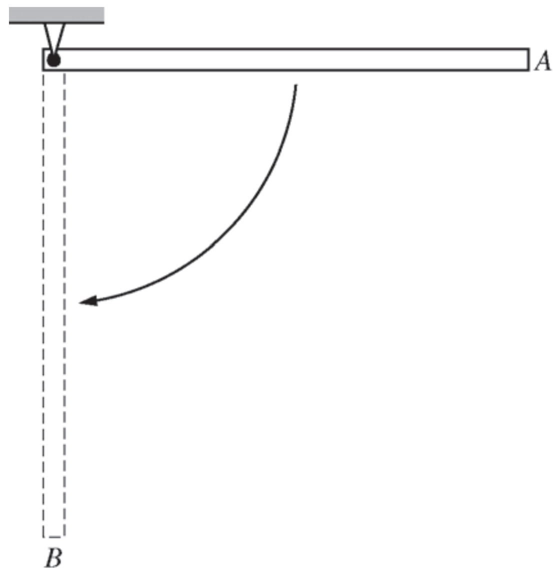
Justify your answer.




Mech.3.

A uniform, thin rod of length $L$ and mass $M$ is allowed to pivot about its end, as shown in the figure above.

(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.




The rod is fixed at one end and allowed to fall from the horizontal position $A$ through the vertical position $B$.

(b) Derive an expression for the velocity of the free end of the rod at position $B$. Express your answer in terms of $M$, $L$, and physical constants, as appropriate.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position $A$ and uses photogates to measure the velocity of the free end at position $B$. The data are recorded below.

| Length (m) | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 |
|---|---|---|---|---|---|---|
| Velocity (m/s) | 2.7 | 3.8 | 4.6 | 5.2 | 5.8 | 6.3 |
| | | | | | | |
| | | | | | | |

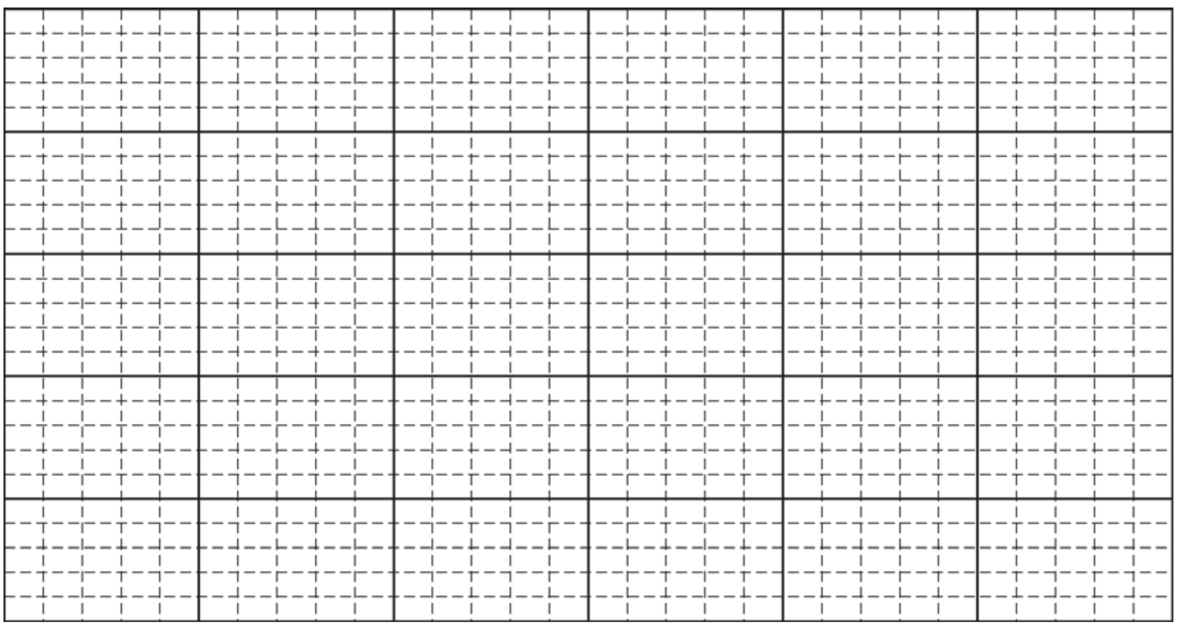
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity $g$.

Horizontal axis: ____________________

Vertical axis: ____________________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

(d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(e)

i. Using your straight line, determine an experimental value for $g$.

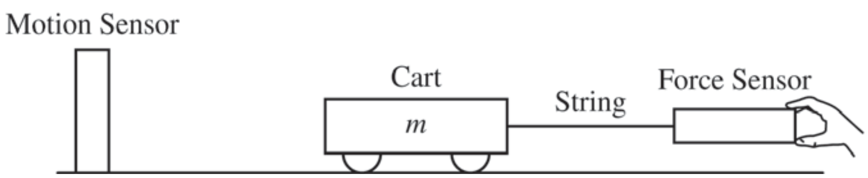
ii. Describe two ways in which the effects of air resistance could be reduced.

**STOP**

**END OF EXAM**

## PHYSICS C: MECHANICS
## SECTION II
## Time—45 minutes
## 3 Questions

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




Mech.1.

A cart of mass $m$ is pulled along a level dynamics track as shown above. A force sensor is attached to the cart with a string and used to measure the horizontal force exerted on the cart to the right. A motion sensor is used to measure the acceleration of the cart with the positive direction toward the right. Friction is not negligible.

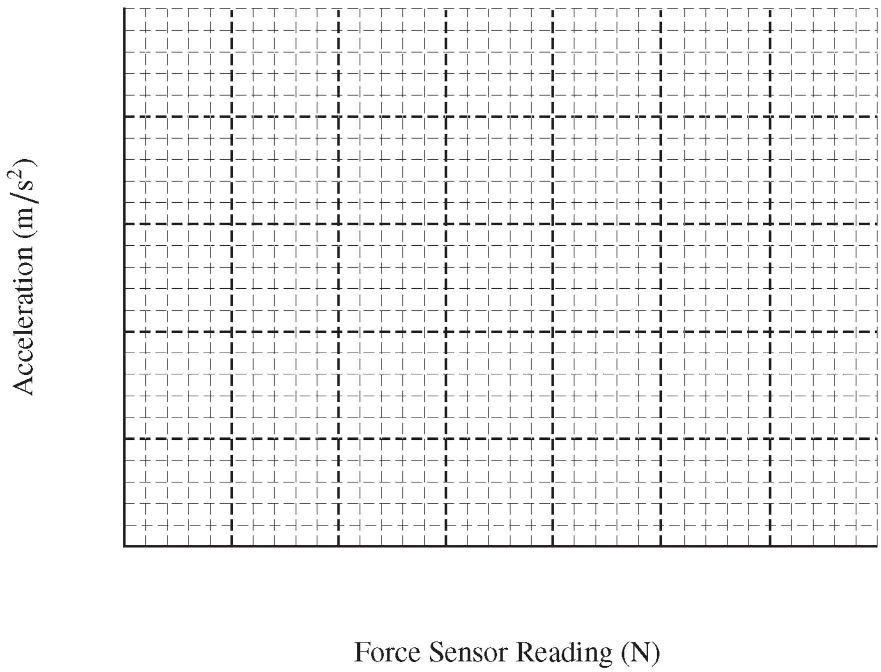
(a) On the dot below, which represents the cart, draw and label the forces (not components) that act on the cart. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

A student pulls the force sensor with a constant force, and the cart accelerates. This is repeated for several trials, with a different constant force used for each trial. The data are recorded in the table below.

| Trial | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Force sensor reading (N) | 0.32 | 0.38 | 0.44 | 0.50 | 0.60 |
| Acceleration $(m/s^2)$ | 0.12 | 0.22 | 0.33 | 0.50 | 0.70 |

(b)

i. On the grid below, plot data points for the acceleration of the cart as a function of the force sensor reading. Clearly scale all axes. Draw a straight line that best represents the data.




ii. Using the straight line from the graph, calculate the mass of the cart.

iii. Using the straight line from the graph, determine the magnitude of the force of friction.

The above experiment is repeated by using a constant force sensor reading of 0.45 N. The cart starts from rest at time $t = 0$ s and is pulled for a time of 2.0 s along the dynamics track.

(c)

i. Determine the acceleration of the cart.

ii. The string breaks at time $t = 2.0$ s. Calculate the time it takes for the cart to stop after the string breaks.

The experiment and analysis in parts (a) and (b) are repeated with a cart that has the same mass but a greater force of friction.

(d)

i. Will the slope of your new line be greater than, less than, or equal to the slope of your line in part (b)i?

____ Greater than ____ Less than ____ Equal to

ii. Will the horizontal intercept of your new line be greater than, less than, or equal to the horizontal intercept of your line in part (b)i?

____ Greater than ____ Less than ____ Equal to


Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where $B$ is a positive constant and $x$ is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed $v_0$ is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.



The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance $D$, as shown above. Express your answers to parts (b), (c), and (d) in terms of $M$, $B$, $v_0$, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance $D$ that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

Justify your answer.





[62]

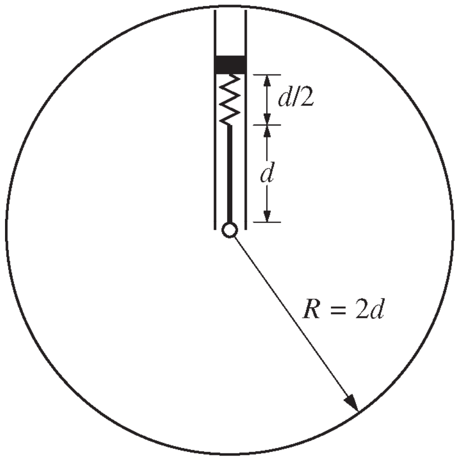



Figure 1

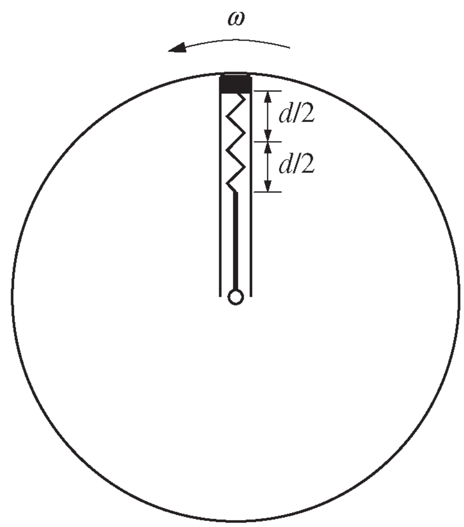



Figure 2

Mech.3.

A uniform rod of length $d$ has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$ . Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$ . Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

| | Mass | Rotational Inertia |
|---|---|---|
| Block | $m$ | |
| Rod | $m_R = 3m$ | $\frac{m_R d^2}{3}$ (about the end of the rod) |
| Platform | $m_P = 5m$ | $\frac{m_P R^2}{2}$ (about the central axis) |

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed $\omega$. Under these conditions, the spring has stretched by an additional length $d/2$ , as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed $\omega$. Express all algebraic answers in terms of $m$, $d$, $\omega$, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

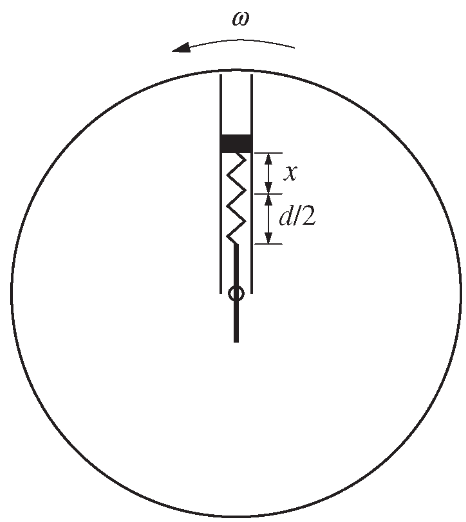



Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed $\omega$ is maintained by the motor driving the platform.

(d) Derive an expression for the distance $x$ that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed $\omega$, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

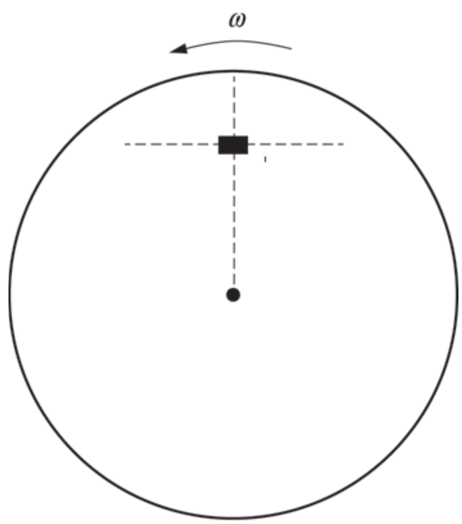


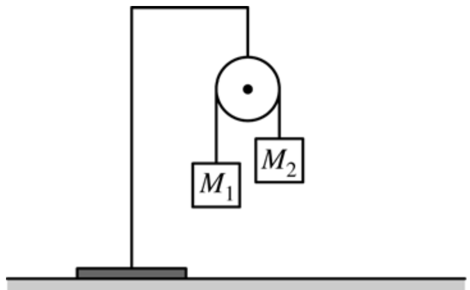

Figure 4

**STOP**

**END OF EXAM**

## PHYSICS C: MECHANICS
### SECTION II
### Time—45 minutes
### 3 Questions

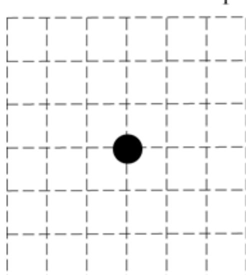
**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



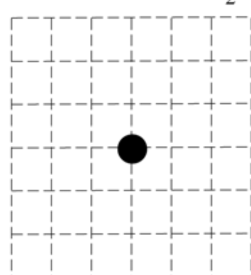

1. An Atwood's machine consists of two blocks connected by a light string that passes over a frictionless pulley of negligible mass, as shown in the figure above. The masses of the two blocks, $M_1$ and $M_2$, can be varied. $M_2$ is always greater than $M_1$.

   (a) On the dots below, which represent the blocks, draw and label the forces (not components) that act on the blocks. Each force must be represented by a distinct arrow starting on and pointing away from the appropriate dot. The relative lengths of the arrows should show the relative magnitudes of the forces.

   Block of Mass $M_1$

   Block of Mass $M_2$

   (b) Using the forces in your diagrams above, write an equation applying Newton's second law to <u>each</u> block and use these two equations to derive the magnitude of the acceleration of the blocks and show that it is given by the equation: $a = \dfrac{(M_2 - M_1)}{(M_1 + M_2)} g$

The magnitude of the acceleration $a$ was measured for different values of $M_1$ and $M_2$, and the data are shown below.

| | | | | | |
|---|---|---|---|---|---|
| $M_1$ (kg) | 1.0 | 2.0 | 5.0 | 6.0 | 10.0 |
| $M_2$ (kg) | 2.0 | 3.0 | 12.0 | 8.0 | 14.0 |
| $a$ (m/s$^2$) | 3.02 | 1.82 | 4.21 | 1.15 | 1.71 |
| | | | | | |
| | | | | | |

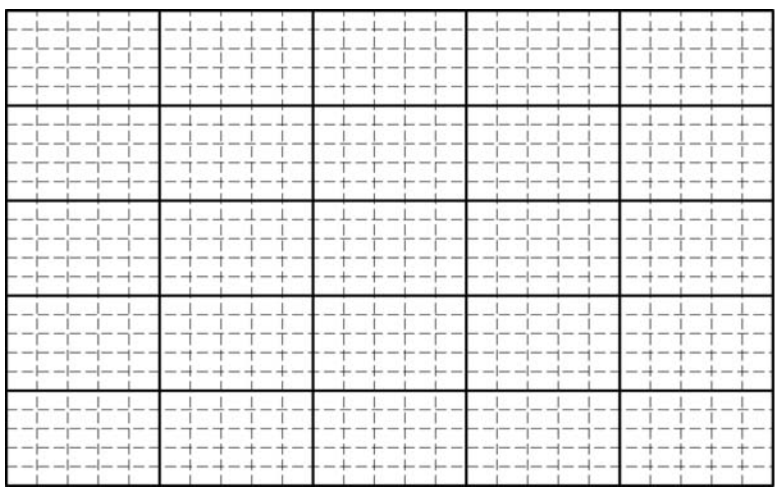
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity $g$.

Vertical axis: ____________________

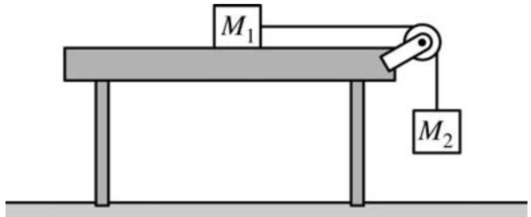
Horizontal axis: ____________________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given.

(d) Plot the data points for the quantities indicated in part (c) on the graph below. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best represents the data.



(e) Using your straight line, determine an experimental value for $g$.




The experiment is now repeated with a modification. The Atwood's machine is now set up so that the block of mass $M_1$ is on a smooth, horizontal table and the block of mass $M_2$ is hanging over the side of the table, as shown in the figure above.

(f) For the same values of $M_1$ and $M_2$, is the magnitude of the tension in the string when the blocks are moving higher, lower, or equal to the magnitude of the tension in the string when the blocks are moving in the first experiment?

____ Higher ____ Lower ____ Equal to

Justify your answer.

(g) The value determined for the acceleration due to gravity $g$ is lower than in the first experiment. Give one physical factor that could account for this lower value and explain how this factor affected the experiment.


Note: Figure not drawn to scale.

2. A block of mass $m$ starts at rest at the top of an inclined plane of height $h$, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant $k$. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of $m$, $h$, $k$, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

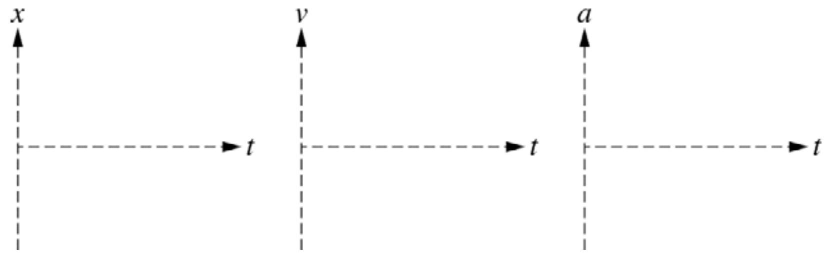
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed $v_0$. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force $F$ is given as a function of speed $v$ by $F = \beta v^2$, where $\beta$ is a positive constant with units of $\text{kg/m}$.

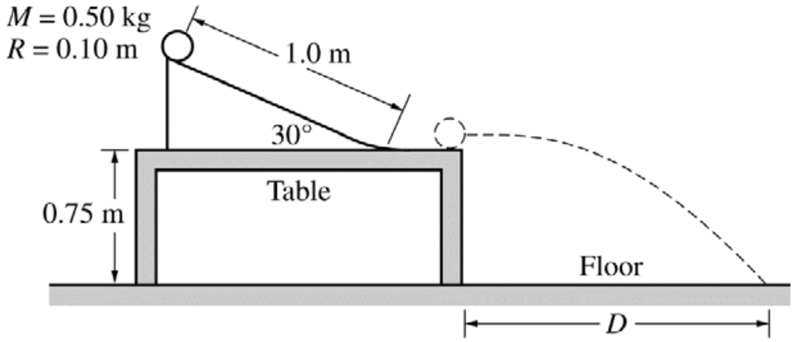
(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time $t$ before it reaches the spring. Express your answer in terms of $m$, $h$, $k$, $\beta$, $v$, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time $t$ can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where $v_0$ is the speed at $t = 0$.

(e) Sketch graphs of position $x$ as a function of time $t$, velocity $v$ as a function of time $t$, and acceleration $a$ as a function of time $t$ for the block as it is moving on the horizontal surface before it reaches the spring.






3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance $D$ away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance $D$.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

**STOP**

**END OF EXAM**

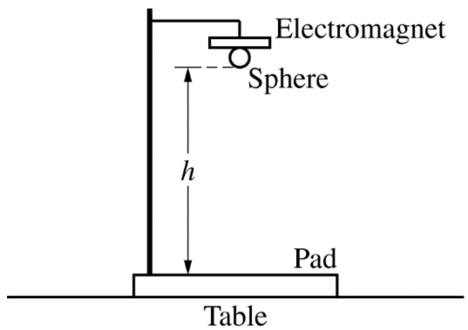
**PHYSICS C: MECHANICS**

**SECTION II**

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




1. A student wants to determine the value of the acceleration due to gravity $g$ for a specific location and sets up the following experiment. A solid sphere is held vertically a distance $h$ above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of $g$ calculated from this one measurement be greater than, less than, or equal to the actual value of $g$ at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance $h$. The student measures and records the time $\Delta t$ of the fall for each particular height, resulting in the following data table.

| $h$ (m) | 0.10 | 0.20 | 0.60 | 0.80 | 1.00 |
|---|---|---|---|---|---|
| $\Delta t$ (s) | 0.105 | 0.213 | 0.342 | 0.401 | 0.451 |
| | | | | | |
| | | | | | |

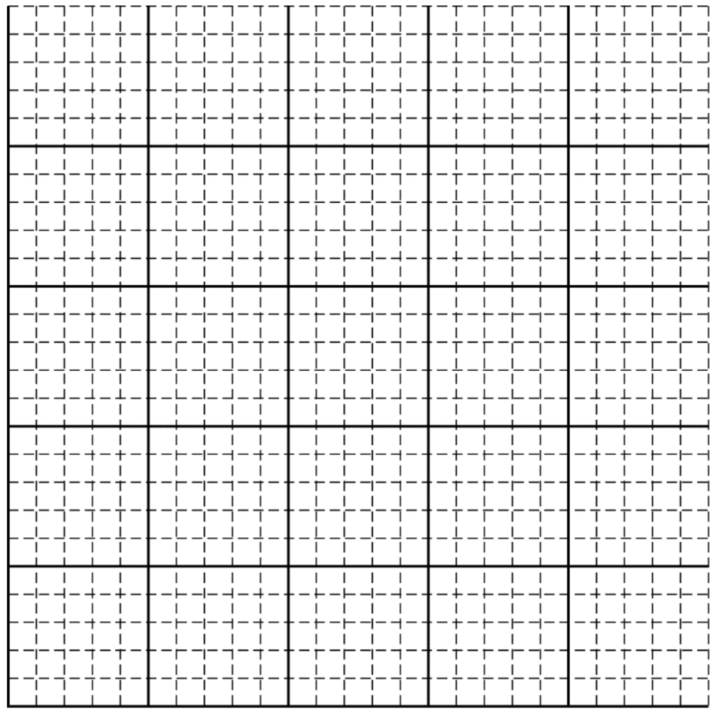
(b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for $g$.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(d) Using the straight line, calculate an experimental value for $g$.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen $y$ as a function of time $t$ is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

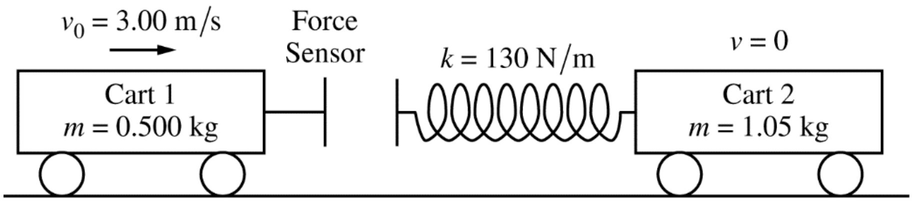
(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

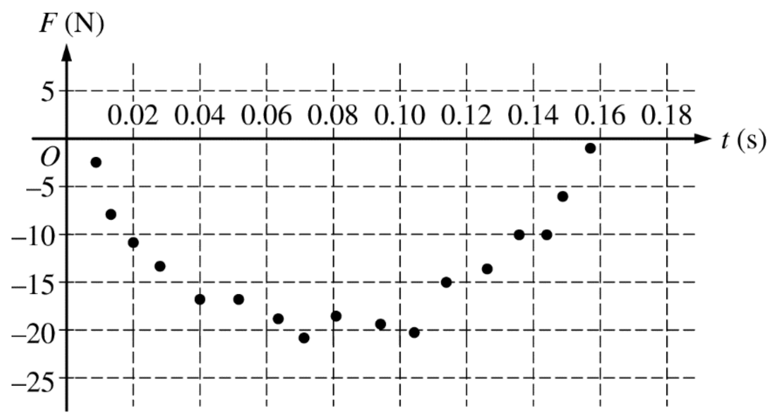
ii. Calculate the new experimental value for $g$.

iii. Using $9.81\ \text{m/s}^2$ as the accepted value for $g$ at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.




2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00 \text{ m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130 \text{ N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200 \text{ N/s}^2\right)t^2 - (500 \text{ N/s})t$ .




(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

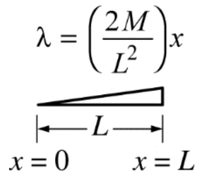
i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.




3. A triangular rod, shown above, has length $L$, mass $M$, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where $x$ is the distance from one end of the rod.

   (a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

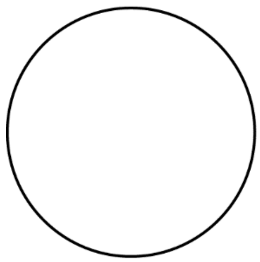


Figure 1

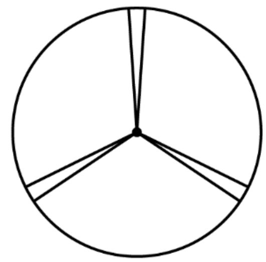


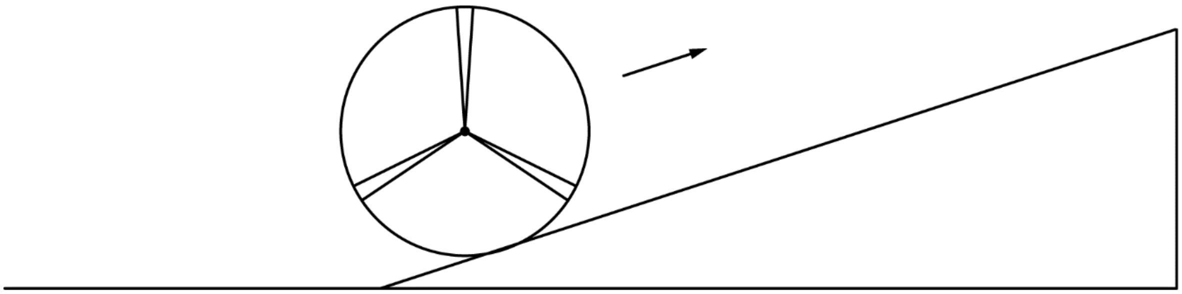
Figure 2

The thin hoop shown above in Figure 1 has a mass $M$, radius $L$, and a rotational inertia around its center of $ML^2$. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia $I_{tot}$ of the hoop-rods system about the center of the hoop. Express your answer in terms of $M$, $L$, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force $F$ is exerted tangent to the hoop for a time $\Delta t$.

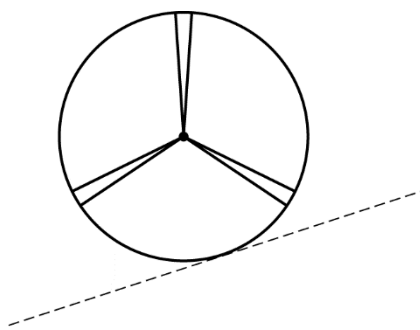
(c) Derive an expression for the final angular speed $\omega$ of the hoop-rods system. Express your answer in terms of $M$, $L$, $F$, $\Delta t$, and physical constants, as appropriate.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed $\omega$ found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of $M$, $L$, $I_{tot}$, $\omega$, and physical constants, as appropriate.

**STOP**

**END OF EXAM**

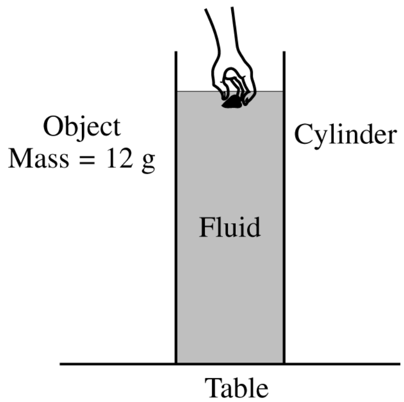
## PHYSICS C: MECHANICS

## SECTION II

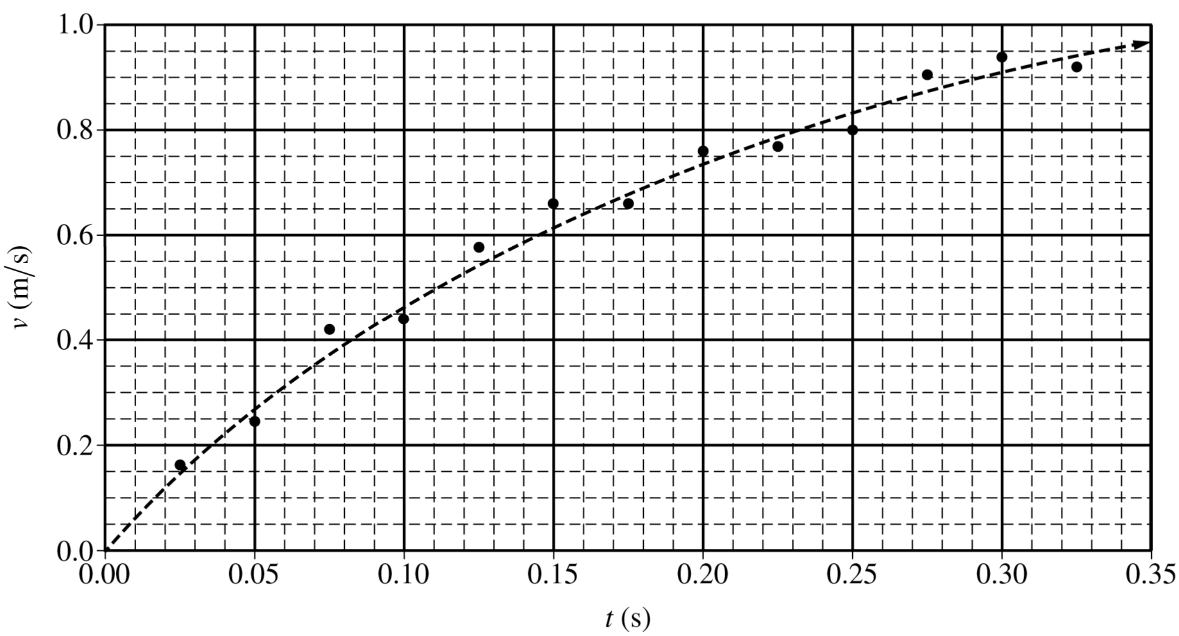
## Time—45 minutes

## 3 Questions

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed $v$ of the falling object as a function of time $t$ are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time $t$.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time $t$.

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

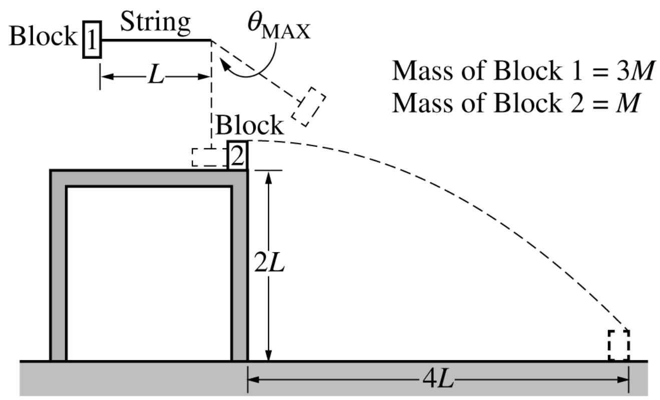
(c)

i. Determine the constant speed of the object.

Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

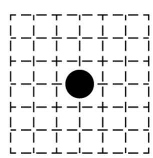



<u>Note</u>: Figure not drawn to scale.

2. A pendulum of length $L$ consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass $M$, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle $\theta_{MAX}$ to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of $M$, $L$, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension $F_T$ in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

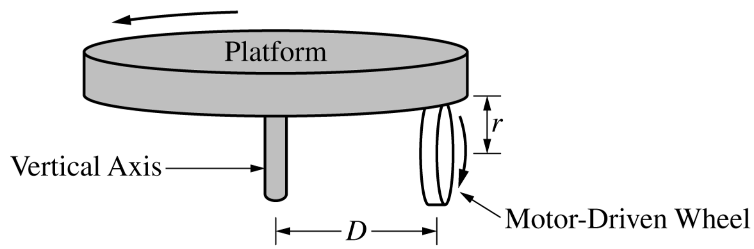
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle $\theta_{MAX}$ that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

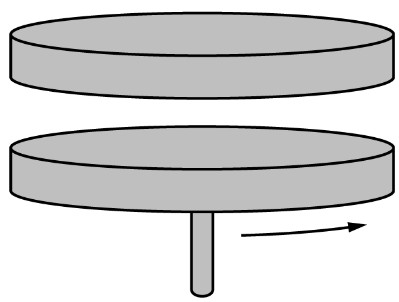



3. A horizontal circular platform with rotational inertia $I_P$ rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius $r$ and touches the platform a distance $D$ from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude $F$ tangent to the wheel until the platform reaches an angular speed $\omega_P$ after time $\Delta t$. During time $\Delta t$, the wheel stays in contact with the platform without slipping.

 (a) Derive an expression for the angular speed $\omega_P$ of the platform. Express your answer in terms of $I_P$, $r$, $D$, $F$, $\Delta t$, and physical constants, as appropriate.

 (b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed $\omega_P$. Express your answer in terms of $I_P$, $r$, $D$, $F$, $\Delta t$, and physical constants, as appropriate.

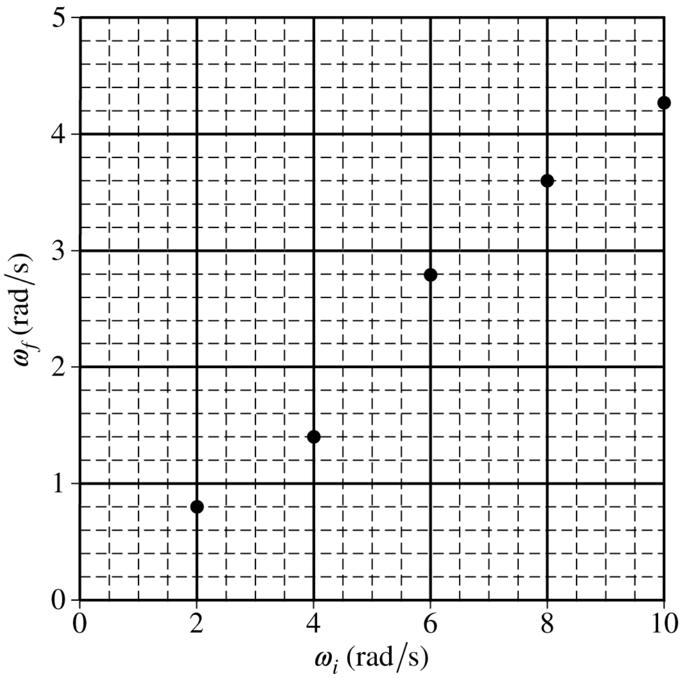
 (c) Derive an expression for the angular speed of the wheel $\omega_W$ when the platform has reached angular speed $\omega_P$. Express your answer in terms of $D$, $r$, $\omega_P$, and physical constants, as appropriate.



When the platform is spinning at angular speed $\omega_P$, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia $I_P$ as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed $\omega_f$.

 (d) Derive an expression for $\omega_f$. Express your answer in terms of $\omega_P$, $I_P$, and physical constants, as appropriate.

A student now uses the rotating platform $\left(I_P = 3.1\ \text{kg}\cdot\text{m}^2\right)$ to determine the rotational inertia $I_U$ of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed $\omega_i$ when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed $\omega_f$. Trials are repeated for different values of $\omega_i$. A graph of $\omega_f$ as a function of $\omega_i$ is shown on the axes below.





[81]

(e)

i. On the graph on the previous page, draw a best-fit line for the data.

ii. Using the straight line, calculate the rotational inertia of the unknown object $I_U$ about a vertical axis passing through its center of mass.

(f) The kinetic energy of the spinning platform before the object is dropped on it is $K_i$. The total kinetic energy of the platform-object system when it reaches angular speed $\omega_f$ is $K_f$. Which of the following expressions is true?

____ $K_f < K_i$ ____ $K_f = K_i$ ____ $K_f > K_i$

Justify your answer.

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of $I_U$ obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

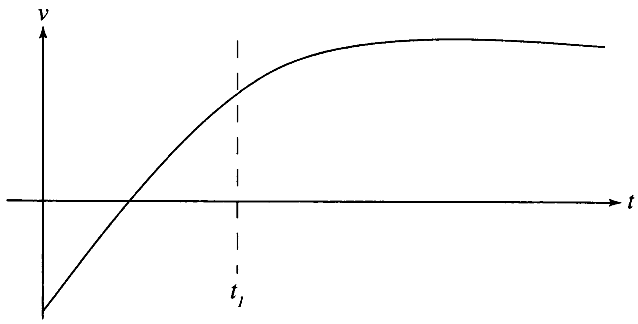
**STOP**

**END OF EXAM**

# Princeton Review Practice Exam

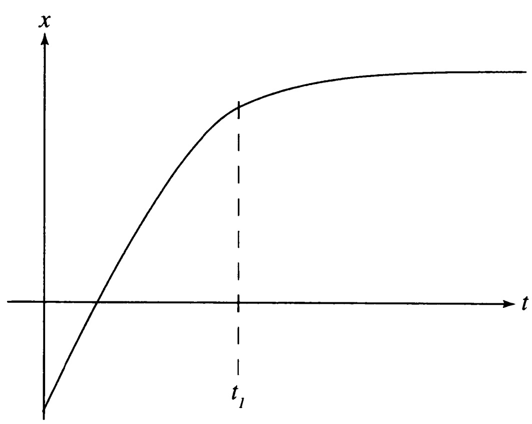
PHYSICS C
SECTION I, MECHANICS
Time—45 minutes
35 Questions

Directions: Each of the following questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then mark it on your answer sheet.

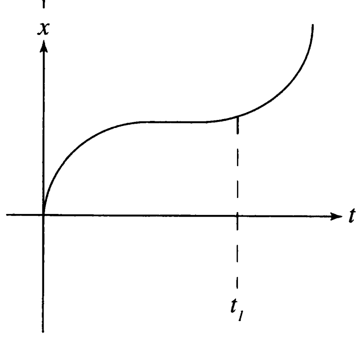



1. The graph above shows the velocity vs. time graph for a 3 kg object moving in one dimension. Which of the following is a possible graph of position-versus-time for this object?

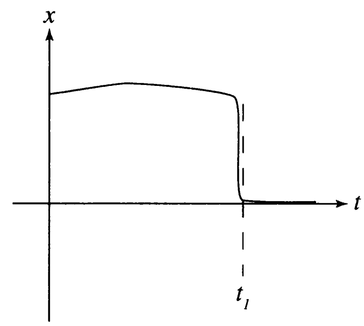
(A)



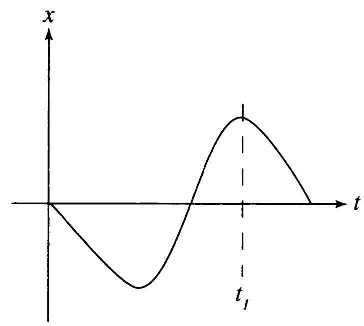

(B)



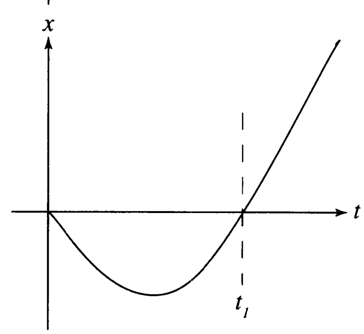

(C)




(D)



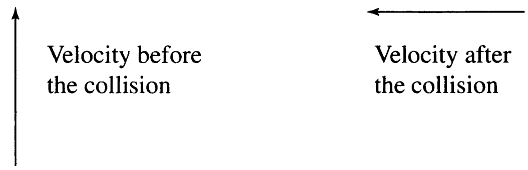

(E)




2. A ball is dropped from an 80 m tall building. How long does the ball take to reach the ground?

(A) 2.8 seconds
(B) 4 seconds
(C) 8 seconds
(D) 8.9 seconds
(E) 16 seconds

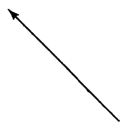
**GO ON TO THE NEXT PAGE.**

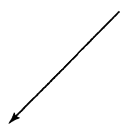


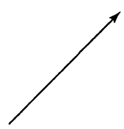

3. The velocity of an object before a collision is directed straight north and the velocity after the collision is directed straight west, as shown above. Which of the following vectors represents the change in momentum of the object?

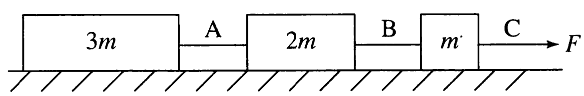
(A) 

(B) 

(C) 

(D) 

(E) 




4. Three blocks of masses $3m$, $2m$, and $m$ are connected to strings A, B, and C as shown above. The blocks are pulled along a frictionless, horizontal floor with a force, $F$. Determine the acceleration of the $2m$ block.

(A) $\frac{F}{2m}$

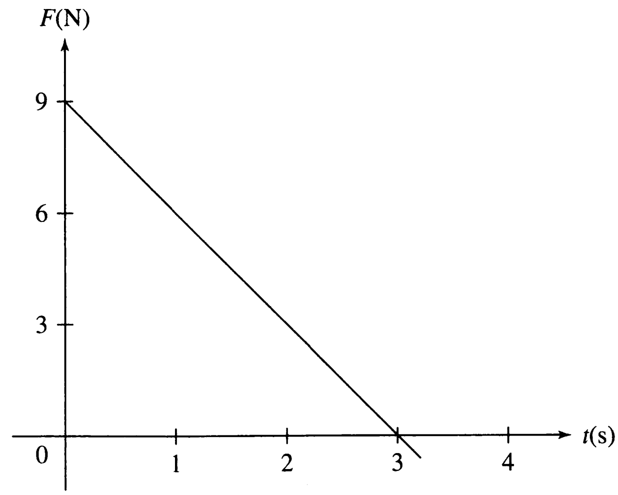
(B) $\frac{F}{6m}$

(C) $2Fm$

(D) $6Fm$

(E) $\frac{F}{m}$

Questions 5-6




A block of mass 2 kg, initially at rest, is pulled along a frictionless, horizontal surface with a force shown as a function of time by the graph above.

5. The acceleration of the block at $t = 2$ s is

(A) 0 m/s²
(B) 1.5 m/s²
(C) 2.0 m/s²
(D) 2.5 m/s²
(E) 3.0 m/s²

6. The speed of the block at $t = 3$ s is

(A) 0 m/s
(B) 4.5 m/s
(C) 6.75 m/s
(D) 13.5 m/s
(E) 54 m/s

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[84]

Questions 7-8


The center of mass of a cylinder of mass $m$, radius $r$, and rotational inertia $I = \frac{1}{2}mr^2$ has a velocity of $v_{cm}$ as it rolls without slipping along a horizontal surface. It then encounters a ramp of angle $\theta$, and continues to roll up the ramp without slipping.

7. What is the maximum height the cylinder reaches?

(A) $\frac{v^2}{4g}$

(B) $\frac{4v^2}{3g}$

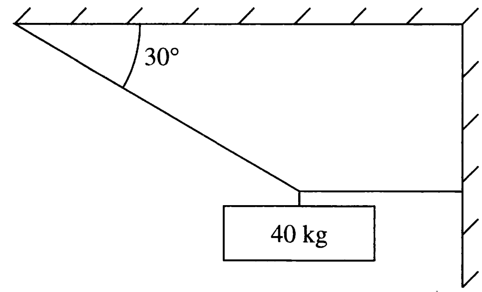
(C) $\frac{v^2}{3g}$

(D) $\frac{3v^2}{4g}$

(E) $\frac{4g}{3v^2}$

8. Now the cylinder is replaced with a hoop that has the same mass and radius. The hoop's rotational inertia is $mr^2$. The center of mass of the hoop has the same velocity as the cylinder when it is rolling along the horizontal surface and the hoop also rolls up the ramp without slipping. How would the maximum height of the hoop compare to the maximum height of the cylinder?

(A) The hoop would reach a greater maximum height than the cylinder.
(B) The hoop and cylinder would reach the same maximum height.
(C) The cylinder would reach a greater maximum height than the hoop.
(D) The cylinder would reach less than half the height of the hoop.
(E) None of the above




9. An object of mass 40 kg is suspended by means of two cords, as shown above. The tension in the angled cord is

(A) 80 N
(B) 230 N
(C) 400 N
(D) 690 N
(E) 800 N

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Questions 10-12

A box is on an incline of angle $\theta$ above the horizontal. The box may be subject to the following forces: frictional ($f$), gravitational ($F_g$), tension from a string connected to it ($F_T$), and normal ($N$). In the following free-body diagrams for the box, the lengths of the vectors are proportional to the magnitudes of the forces.

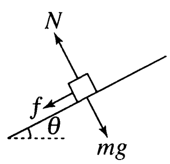


Figure A

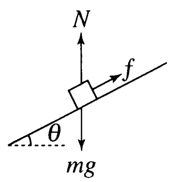


Figure B

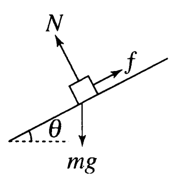


Figure C

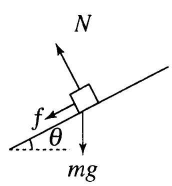


Figure D

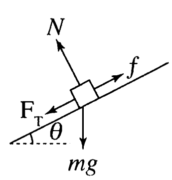


Figure E

10. Which of the following best represents the free-body diagram for the box if it is decelerating as it goes up the incline?

(A) Figure A
(B) Figure B
(C) Figure C
(D) Figure D
(E) Figure E

11. Which of the following best represents the free-body diagram for the box if it is moving at a constant velocity down the ramp?

A) Figure A
(B) Figure B
(C) Figure C
(D) Figure D
(E) Figure E

12. Which of the following best represents the free-body diagram for the box if its speed is increasing as it moves down the incline?

(A) Figure A
(B) Figure B
(C) Figure C
(D) Figure D
(E) Figure E

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13. The force on an object as a function of time $t$ is given by the expression $F = Ct^3$, where $C$ is a constant. Determine the change in momentum for the time interval 0 to $t_1$.

(A) $\dfrac{Ct_1^2}{2}$

(B) $\dfrac{Ct_1^3}{3}$

(C) $\dfrac{Ct_1^4}{3}$

(D) $\dfrac{Ct_1^3}{4}$

(E) $\dfrac{Ct_1^4}{4}$

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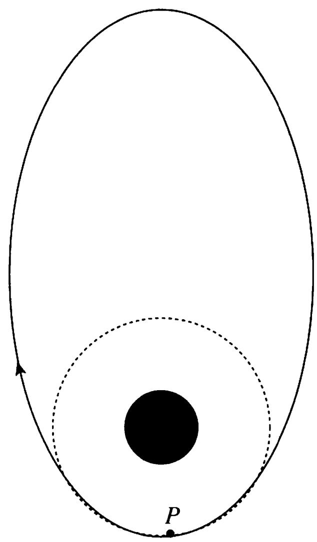
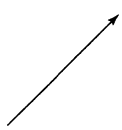
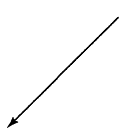



14. A spaceship orbits Earth in a clockwise, elliptical orbit as shown above. The spaceship needs to change to a circular orbit. When the spaceship passes point $P$, a short burst of the ship's engine will change its orbit. What direction should the engine burst be directed?

(A) 

(B) 

(C) 

(D) 

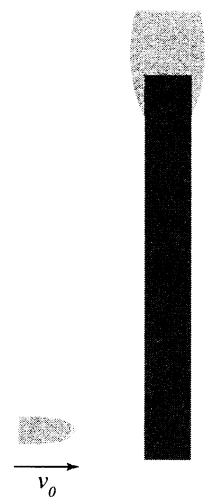
(E) 

15. A motorcycle of mass 200 kg completes a vertical, circular loop of radius 5 m, with a constant speed of 10 m/s. How much work is done on the motorcycle by the normal force of the track?

(A) 0 J
(B) $1 \times 10^5$ J
(C) $1 \times 10^6$ J
(D) 4 J
(E) $10\pi$ J

16. A ball with a radius of 0.2 m rolls without slipping on a level surface. The center of mass of the ball moves at a constant velocity, moving a distance of 30 meters in 10 seconds. The angular speed of the ball about its point of contact on the surface is

(A) 0.6 rad/s
(B) 3 rad/s
(C) 8 rad/s
(D) 15 rad/s
(E) 60 rad/s




17. A bullet is moving with a velocity $v_0$ when it collides with and becomes embedded in a wooden bar that is hinged at one end, as shown above. Consider the bullet and the wooden bar to be the system. For this scenario, which of the following is true?

(A) The linear momentum of the system is conserved because the net force on the system is zero.
(B) The angular momentum of the system is conserved because the net torque on the system is zero.
(C) The kinetic energy of the system is conserved because it is an inelastic collision.
(D) The kinetic energy of the system is conserved because it is an elastic collision.
(E) Linear momentum and angular momentum are both conserved.

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Questions 18-19

A spring mass system is vibrating along a frictionless, horizontal floor. The spring constant is 8 N/m, the amplitude is 5 cm, and the period is 4 seconds.

18. In kg, the mass of the system is
    (A) $32\pi^2$
    (B) $\frac{32}{\pi^2}$
    (C) $\frac{16}{\pi}$
    (D) $\frac{0.2}{\pi^2}$
    (E) $\frac{20}{\pi^2}$

19. Which of the following equations could represent the position of the mass from equilibrium $x$ as a function of time $t$, where $x$ is in meters and $t$ is in seconds.
    (A) $x = 0.05 \cos \pi t$
    (B) $x = 0.05 \cos 2\pi t$
    (C) $x = 0.05\cos\frac{\pi}{2}t$
    (D) $x = 8\cos\frac{\pi}{2}t$
    (E) $x = 0.05\cos\frac{\pi}{4}t$

20. Two blocks of masses $M$ and $3M$ are connected by a light string. The string passes over a frictionless pulley of negligible mass so that the blocks hang vertically. The blocks are then released from rest. What is the acceleration of the mass $M$?
    (A) $\frac{g}{4}$
    (B) $\frac{g}{3}$
    (C) $g$
    (D) $\frac{2g}{3}$
    (E) $\frac{g}{2}$

21. For a particular nonlinear spring, the relationship between the magnitude of the applied force, $F$, and the stretch of the spring, $x$, is given by the equation $F = kx^{1.5}$. How much energy is stored in the spring when is it stretched a distance $x_1$?
    (A) $\frac{2kx_1^{2.5}}{5}$
    (B) $\frac{kx_1^{1.5}}{5}$
    (C) $kx_1^{2.5}$
    (D) $\frac{1}{2}kx_1^{2}$
    (E) $1.5kx_1^{.5}$

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Questions 22-23

Two ice skaters are moving on frictionless ice and are about to collide. The 50-kg skater is moving directly west at 4 m/s. The 75-kg skater is moving directly north at 2 m/s. After the collision they stick together.

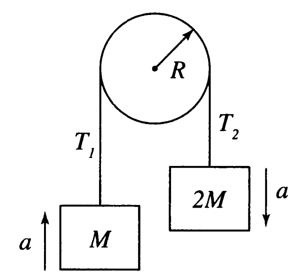
22. What is the magnitude of the momentum of the two-skater system after the collision?
    (A) 50 kg•m/s
    (B) 150 kg•m/s
    (C) 200 kg•m/s
    (D) 250 kg•m/s
    (E) 350 kg•m/s

23. For this scenario, which of the following is true?
    (A) The linear momentum of the system is conserved because the net force on the system is nonzero during the collision.
    (B) Only the kinetic energy of the system is conserved because it is an inelastic collision.
    (C) Only the kinetic energy of the system is conserved because it is an elastic collision.
    (D) The linear momentum of the system is conserved because the net force on the system is zero.
    (E) Both the linear momentum and the kinetic energy of the system are conserved.

---

24. The position of an object is given by the equation $x = 2.0t^3 + 4.0t + 6.25$, where $x$ is in meters and $t$ is in seconds. What is the acceleration of the object at $t = 1.50$ s?
    (A) 6 m/s²
    (B) 12 m/s²
    (C) 18 m/s²
    (D) 24 m/s²
    (E) 32 m/s²

25. An astronaut in space accidentally becomes disconnected from her ship's tether. In order to get back to safety, she throws a wrench of mass $m = 2$ kg directly away from the ship at a speed $v = 30$ m/s. Given that she has a mass of 60 kg and was at rest before throwing the wrench, how long will it take her to get back to the ship if she is 35 m away from it?
    (A) 60 s
    (B) 45 s
    (C) 35 s
    (D) 25 s
    (E) 15 s

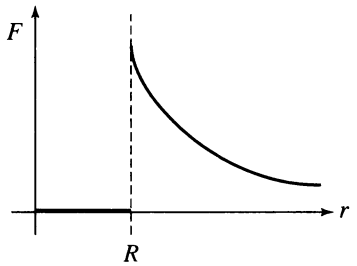



26. Two blocks of masses $M$ and $2M$ are connected by a light string. The string passes over a pulley, as shown above. The pulley has a radius $R$ and moment of inertia $I$ about its center. $T_1$ and $T_2$ are the tensions in the string on each side of the pulley and $a$ is the acceleration of the masses. Which of the following equations best describes the pulley's rotational motion during the time the blocks accelerate?
    (A) $(T_2 + T_1)R = Ia$
    (B) $(T_2 - T_1)R = Ia$
    (C) $(T_2 - T_1)R = I\frac{a}{R}$
    (D) $MgR = I\frac{a}{R}$
    (E) $3MgR = I\frac{a}{R}$

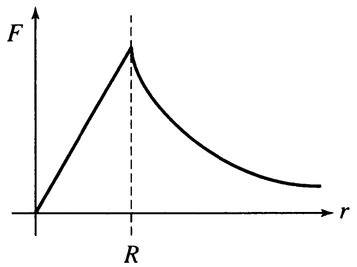
**GO ON TO THE NEXT PAGE.**

27. A solid sphere of uniform density with mass $M$ and radius $R$ is located far out in space. A test mass, $m$, is placed at various locations both within the sphere and outside the sphere. Which graph correctly shows the force of gravity on the test mass vs. the distance from the center of the sphere?

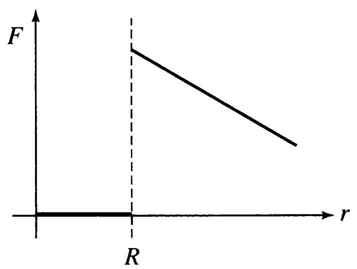
(A)



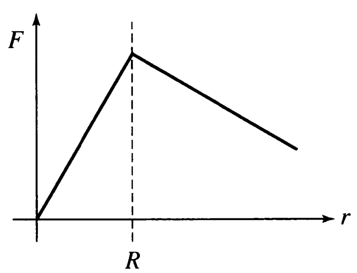

(B)



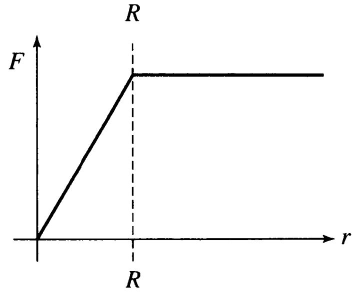

(C)




(D)




(E)




28. The Gravitron is a carnival ride that looks like a large cylinder. People stand inside the cylinder against the wall as it begins to spin. Eventually, it is rotating fast enough that the floor can be removed without anyone falling. Given then the coefficient of friction between a person's clothing and the wall is $\mu$, the tangential speed is $v$, and the radius of the ride is $r$, what is greatest mass that a person can be to safely go on this ride?

(A) $\mu v^2/(rg)$
(B) $\mu v^2/(r^2 g)$
(C) $r^2 v^2/(\mu g)$
(D) $rg/(\mu v^2)$
(E) None of the above

29. A simple pendulum of length $L$, mass $m$, and amplitude $A$ has a frequency of $f$ on Earth. If this pendulum were moved to the Moon (which has $\frac{1}{6}$ Earth's gravity), what would be its new frequency?

(A) $\left(\frac{1}{6}\right)^2 f$

(B) $\left(\frac{1}{6}\right)^{\frac{1}{2}} f$

(C) $(6^2)f$

(D) $(6)^{\frac{1}{2}} f$

(E) $f$

30. An electric car of mass 300 kg delivers 400 W as it moves the car at a constant 20 m/s. The force delivered by the motor is

(A) $\frac{4}{3}$ N
(B) 20 N
(C) 600 N
(D) 6,000 N
(E) 8,000 N

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31. A 2.0 kg mass is attached to the end of a vertical ideal spring with a spring constant of 800 N/m. The mass is pulled down 10 cm from the equilibrium position and then released, so that it oscillates. The kinetic energy of the 2.0 kg mass at the equilibrium position is

(A) $\frac{2}{3}$ J

(B) 2 J

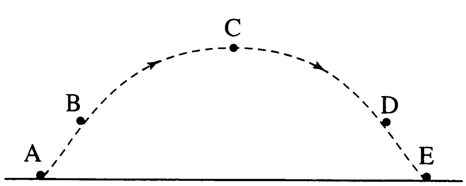
(C) 4 J

(D) 12 J

(E) 40 J

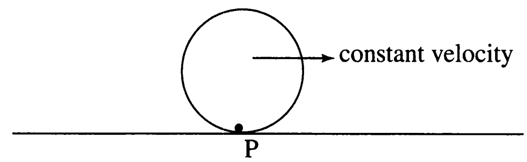
32. Physics students are checking the constant acceleration equations of kinematics by measuring the velocity of a tennis ball that is dropped and falls 6 meters and then passes through a photogate. The predicted velocity is 20% above the velocity measured by the photogate. Which of the following best describes the cause of the large percent difference?

(A) The ball changes its shape while falling.
(B) The acceleration of gravity varies as the ball is falling.
(C) Air resistance increases the acceleration of the ball.
(D) The acceleration of the balls varies with the velocity.
(E) The acceleration of gravity changes due to air resistance.




33. An object is launched and follows the dashed path shown above. If air resistance is considered, when is the velocity of the object the greatest and the acceleration of the object the greatest?

| | Greatest Velocity | Greatest Acceleration |
|---|---|---|
| (A) | A | All equal to $g$ |
| (B) | C | All equal to $g$ |
| (C) | A | A |
| (D) | E | E |
| (E) | A | E |




34. A disk is rolling without slipping along the ground and the center of mass is traveling at a constant velocity, as shown above. What direction is the acceleration of the contact point P and the center of mass?

| | Acceleration of Contact Point P | Acceleration of Center of Mass |
|---|---|---|
| (A) | Upward | To the right |
| (B) | Upward | Zero |
| (C) | To the right | Zero |
| (D) | To the right | To the right |
| (E) | Upward and to the right | Zero |

35. The escape velocity for a rocket launched from the surface of a planet is $v_0$. Determine the escape velocity for another planet that has twice the mass and twice the radius of this planet.

(A) $2v_0$

(B) $\frac{1}{2}v_0$

(C) $\frac{1}{\sqrt{2}}v_0$

(D) $\sqrt{2}v_0$

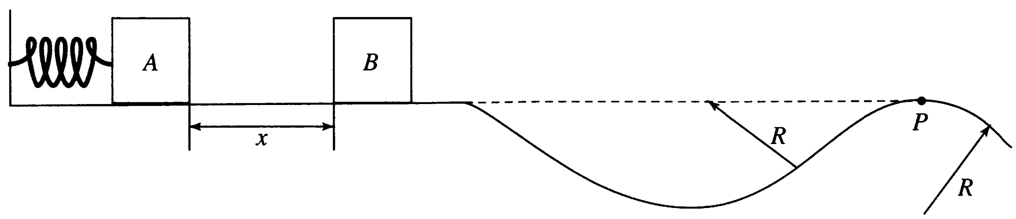
(E) $v_0$

## STOP

END OF SECTION I, MECHANICS

PHYSICS C
SECTION II, MECHANICS
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes per question for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight.




1. A massless spring with force constant $k$ is attached at its left end to a wall, as shown above. Initially, block $A$ and block $B$, each of mass $M$, are at rest on a frictionless, level surface, with block $A$ in contact with the spring (but not compressing it) and block $B$ a distance $x$ from block $A$. Block $A$ is then moved to the left, compressing the spring a distance of $d$, and held in place while block $B$ remains at rest. First block $A$ is released, then as it passes the equilibrium position loses contact with the spring. After block $A$ is released it moves forward and has a perfectly inelastic collision with block $B$ and then follows the frictionless, curved path shown above. The radius of the valley and the hill in the diagram are both $R$. Answer the following in terms of $M$, $k$, $d$, $x$, $g$, and $R$.

   (a) Determine the speed of block $A$ just before it collides with block $B$.
   (b) Determine the speed of block $B$ just after the collision occurs.
   (c) Determine the change in kinetic energy for the collision.
   (d) Determine the normal force on the boxes when they are at position $P$, the top of the hill.

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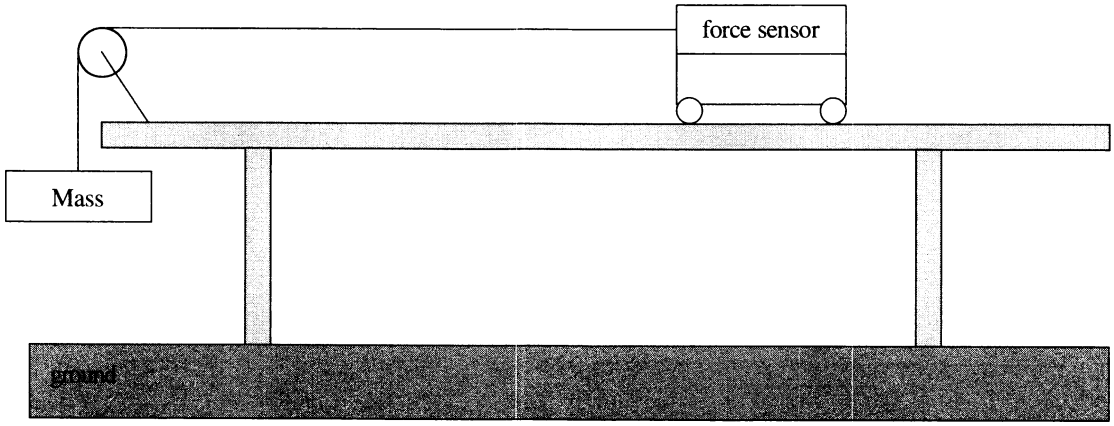
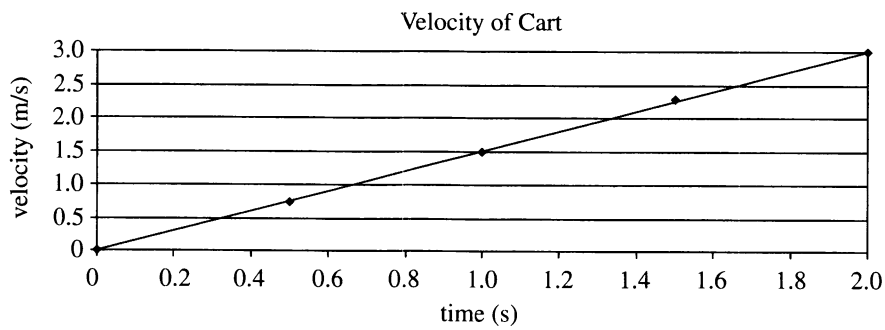



2. Physics students are performing a lab using the mass-pulley system shown above. The cart has a force sensor attached to it, and the total mass of the force sensor and cart is 1.2 kg. The force sensor is attached to a 300-gram hanging mass by a string that is placed over a pulley. When the system is released, a motion detector recording the motion of the cart produces the following velocity versus time graph.




(a) Calculate the acceleration of the cart.

(b) Assume tension is the only horizontal force on the force sensor/cart combination (ignore friction). Calculate the tension in the string for this scenario.

(c) The measured tension is actually 20% greater than the tension predicted in (b). Explain why this might be the case. The force sensor and the motion sensor are working properly, so do not use faulty data to explain the result.

(d) Describe a process you could use to determine the rotational inertia of the pulley with this system, a meterstick, the force sensor, and motion sensor.

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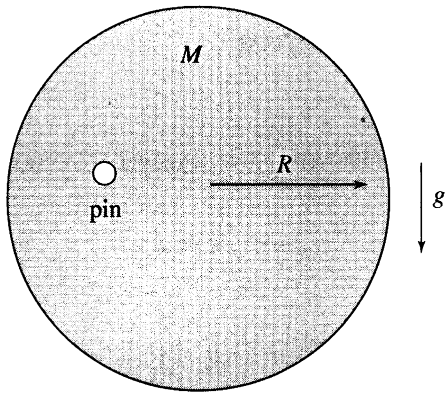



Figure I

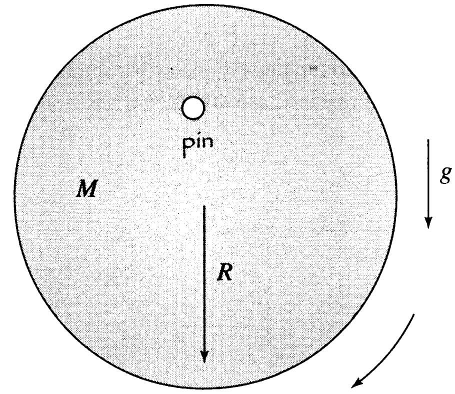



Figure II

3. A disk of mass $M$ and radius $R$ is pinned half of the way along its radius, and held in a horizontal position, as shown in Figure I. The rotational inertia of the disk about its center is $\frac{1}{2}MR^2$. The disk is released at $t = 0$ s, and falls to the vertical position shown in Figure II, and it continues to rotate about the pin. Answer the following in terms of $M$, $R$, and $g$.

(a) Calculate the rotational inertia of the disk about the pin.
(b) Calculate the angular acceleration of the disk at $t = 0$ s.
(c) Calculate angular velocity of the disk when it is in the vertical position shown in Figure II.

Now the disk is stopped and brought to rest in the vertical position shown in Figure II. It is given a slight disturbance to an angle $\theta_0$.

(d) Calculate the angular frequency of the oscillation.

STOP

END OF SECTION II, MECHANICS

# Mooney Practice Exam

## AP PHYSICS C TEST

SECTION I–MECHANICS

Time: 45 minutes

35 Questions

**Directions:** Each of the questions or incomplete statements below is followed by 5 suggested answers or completions. Select the one that is best in each case.

**Note:** To simplify calculations, you may use $g = 10$ m/s$^2$ in all calculations

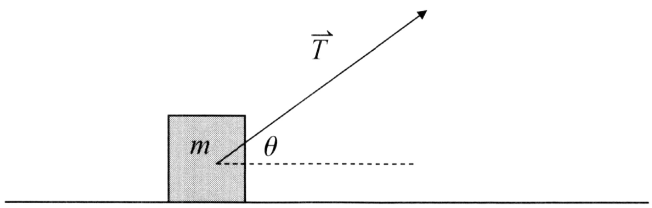
1. A 20-kg mass moving at 10 m/s decelerates uniformly to rest in 5 s. During this time, the mass has traveled a distance of

(A) 2 m (B) 50 m (C) 25 m (D) 125 m (E) 12.5 m

2. A 2-kg mass is projected straight up with an initial speed of 20 m/s. In describing the velocity and acceleration of the object, a student chooses the starting point as the origin and the positive direction as up. At the highest point of the projectile, how will the student describe the velocity and acceleration of the mass?

| | Velocity | Acceleration |
|---|---|---|
| (A) | negative | positive |
| (B) | positive | negative |
| (C) | 0 | 0 |
| (D) | 0 | positive |
| (E) | 0 | negative |

**Questions 3 and 4**




A mass $m$ is pulled along a rough surface at a constant speed by a force maintained at an angle $\theta$ with the horizontal.

3. The work done by $\vec{T}$ as the mass moves a distance $D$ is

(A) 0 (B) $TD\cos\theta$ (C) $\frac{T}{m}\cos\theta$ (D) $TD\sin\theta$ (E) $\frac{T}{m}\sin\theta$

4. The coefficient of friction is

(A) $\tan\theta$ (B) $\frac{T\cos\theta}{mg - T\sin\theta}$ (C) $\frac{T\cos\theta}{T\sin\theta + mg}$ (D) $\frac{T\sin\theta}{mg - T\cos\theta}$ (E) $\frac{T\sin\theta}{T\cos\theta + mg}$

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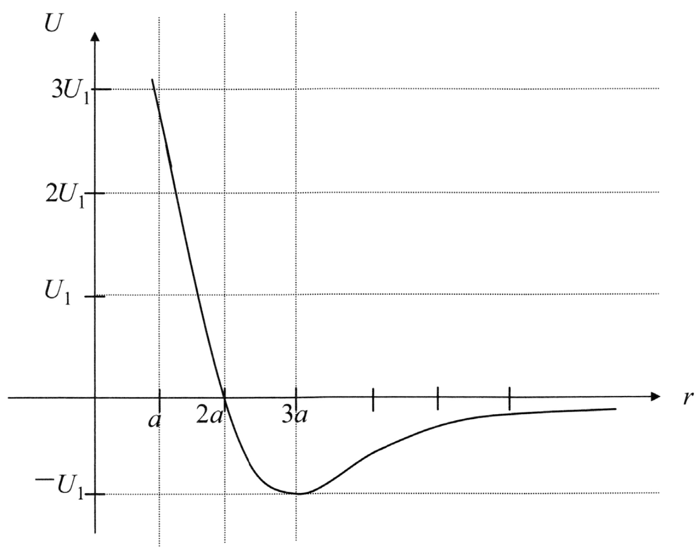
5. A satellite of mass $M$ moves with speed v in a circular orbit of radius $R$ around Earth. Which of the following is true?
   I. To place a larger mass in the same orbit would require a larger orbit speed.
   II. The centripetal acceleration is independent of the mass of Earth.
   III. The angular momentum of the satellite is constant.

   (A) I only (B) I and III only (C) I and II only (D) III only (E) II and III only

6. A mass $m$ falls from rest and experiences an air resistance force of the form $F_{air} = -bv^2$. The terminal speed of the mass is

   (A) $\frac{mg}{b}$ (B) $\sqrt{\frac{mg}{b}}$ (C) $\sqrt{\frac{b}{mg}}$ (D) $\sqrt{\frac{g}{b}}$ (E) $\frac{g}{b}$

**Questions 7 and 8**




A mass $m$ moves under the influence of a potential energy function given by the graph above.

7. At the point $r = 2a$, the force exerted on the mass is closest to

   (A) 0 (B) $\frac{U_1}{a}$ (C) $\frac{4U_1}{a}$ (D) $\frac{3U_1}{a}$ (E) $\frac{3U_1}{2a}$

8. The maximum speed of a mass released from rest at the point $r = a$ is

   (A) $\sqrt{\frac{U_1}{m}}$ (B) $\sqrt{\frac{2U_1}{m}}$ (C) $\sqrt{\frac{4U_1}{m}}$ (D) $\sqrt{\frac{6U_1}{m}}$ (E) $\sqrt{\frac{8U_1}{m}}$

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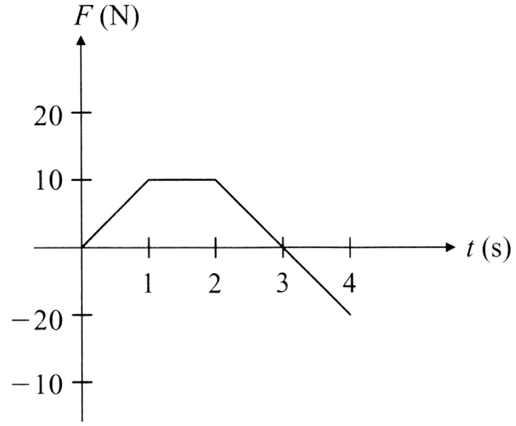
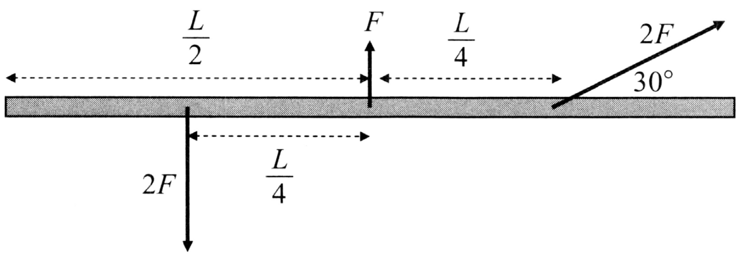



9. An object experiences the nonconstant net force shown in the figure. The change in momentum of the object over the 4 seconds that the force acts is closest to

(A) $10 \frac{\text{kg} \cdot \text{m}}{\text{s}}$ (B) $15 \frac{\text{kg} \cdot \text{m}}{\text{s}}$ (C) $20 \frac{\text{kg} \cdot \text{m}}{\text{s}}$ (D) $25 \frac{\text{kg} \cdot \text{m}}{\text{s}}$ (E) $30 \frac{\text{kg} \cdot \text{m}}{\text{s}}$

10. A mass $6m$ moving with speed $+v$ across a smooth horizontal surface explodes into three pieces of masses $m$, $2m$, and $3m$, respectively. After the explosion, the mass $m$ is stationary, and the mass $2m$ is moving with velocity $+2v$. What is the velocity of the other mass?

(A) $\frac{1}{2}v$ (B) $\frac{1}{3}v$ (C) 0 (D) $\frac{2}{3}v$ (E) $\frac{3}{2}v$




11. Three forces act on a light stick as indicated in the figure. The net torque about the center of the stick is

(A) 0 (B) $\frac{1}{2}FL$ (C) $\frac{3}{4}FL$ (D) $FL$ (E) $2FL$

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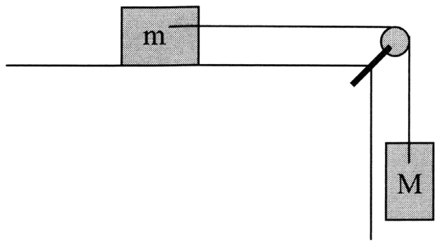


12. A mass $m$ is pulled across a frictionless horizontal surface by a string attached to a descending mass $M$ over a light, frictionless pulley. The tension in the string is

(A) $Mg$ (B) $\frac{mM}{m+M}g$ (C) $\frac{m}{M}g$ (D) $\frac{M}{m}g$ (E) $\frac{mM}{M-m}g$

13. A satellite of mass $m$ moves in a circular orbit of radius $R$ about the Earth. The angular momentum of the satellite about the center of the Earth has a magnitude

(A) $m\sqrt{\frac{M_eR}{G}}$ (B) $m\sqrt{\frac{GR}{M_e}}$ (C) $m\sqrt{\frac{GM_e}{R}}$ (D) $m\sqrt{GM_eR}$ (E) $m\sqrt{\frac{R}{GM_e}}$

14. A physical pendulum oscillates through small angles about the vertical with the angle, measured in radians, obeying the differential equation $\frac{d^2\theta}{dt^2} = -4\pi\theta$. The period of the oscillations in seconds is

(A) $\frac{1}{8\pi}$ (B) $8\pi$ (C) $\frac{1}{4}$ (D) $\sqrt{\pi}$ (E) $4\pi$

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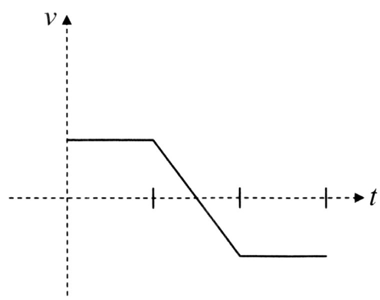
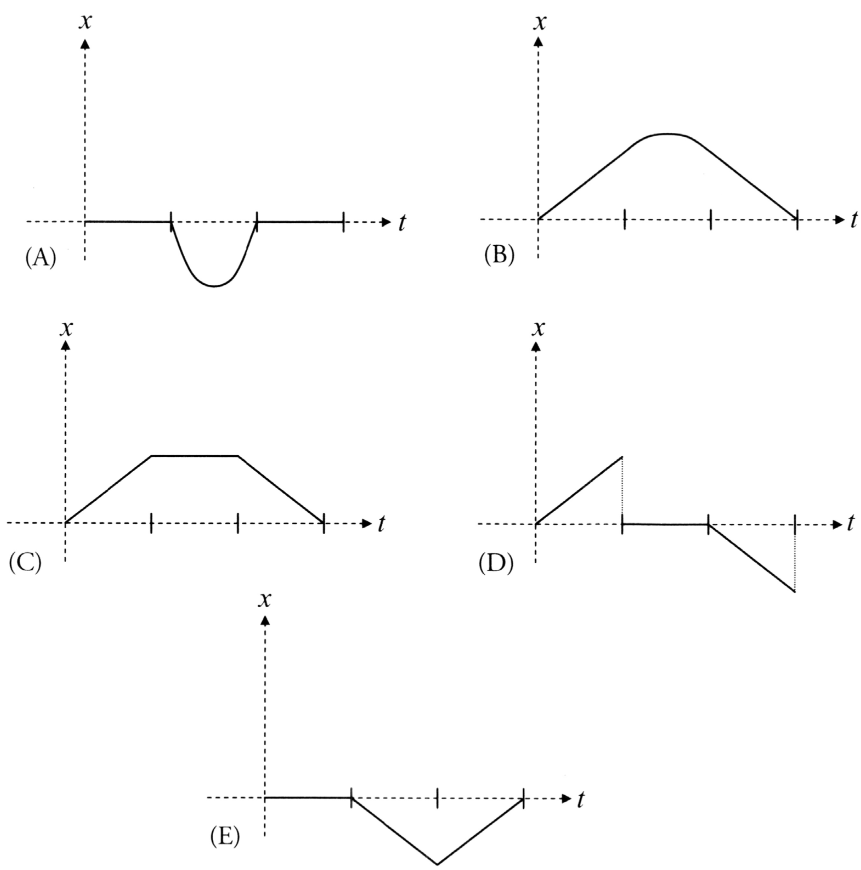



15. An object moves in one dimension with a velocity vs. time graph as shown above. The graph that best represents the displacement of the object over the same time interval is which of the following?

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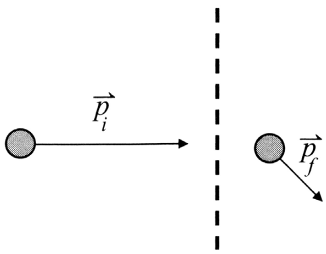
16. A mass $m$ is projected with a velocity of magnitude 50 m/s, making an angle of 37° with the horizontal, the $x$-axis. At the highest point, which of the following could be values for the magnitude of the velocity components and the vertical acceleration component?

| | $v_x$ | $v_y$ | $a_y$ |
|---|---|---|---|
| (A) | 30 m/s | 40 m/s | 10 m/s$^2$ |
| (B) | 40 m/s | 0 | 10 m/s$^2$ |
| (C) | 30 m/s | 0 | 10 m/s$^2$ |
| (D) | 40 m/s | 0 | 0 |
| (E) | 40 m/s | 30 m/s | 10 m/s$^2$ |




17. A mass with momentum $\vec{p}_i$ is acted upon by a force, causing the momentum to change to $\vec{p}_f$. The vector that best represents the impulse delivered by the force during the change is

(A) ↖ (B) ↗ (C) ↙ (D) ↘ (E) ←

**Questions 18 and 19**

A 4 kg mass oscillates on a smooth surface at the end of a horizontal spring according to the equation $x(t) = 0.5\cos\left(6\pi t + \frac{\pi}{8}\right)$, with $x$ measured in meters.

18. The value of the spring constant is

(A) $24\pi$ (B) $\frac{\pi^2}{4}$ (C) $\frac{\pi}{2}$ (D) $144\pi^2$ (E) $36\pi^2$

19. The maximum acceleration of the mass is

(A) $3\pi$ (B) $18\pi^2$ (C) $\frac{\pi}{16}$ (D) $\frac{\pi^2}{128}$ (E) $\frac{9}{2}\pi^2$

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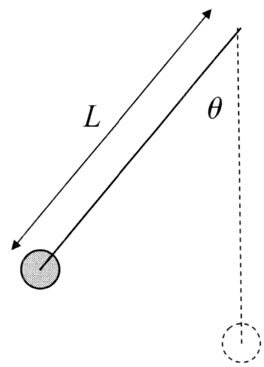
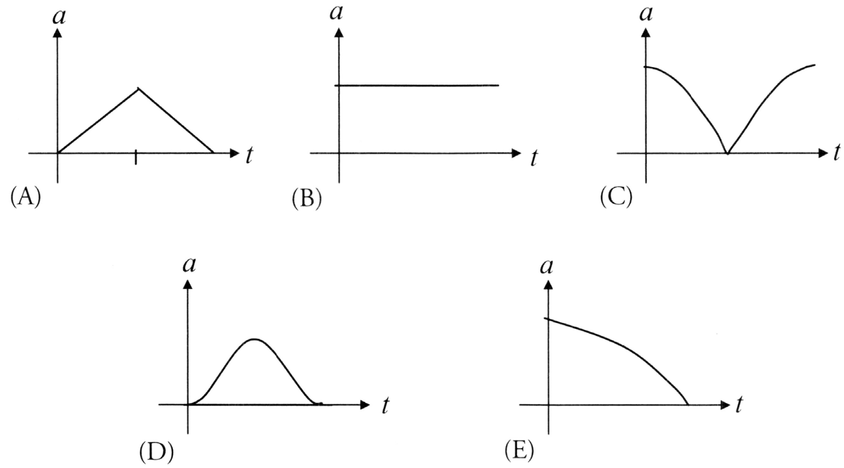
## Questions 20–22




A simple pendulum initially making an angle $\theta$ with the vertical is released from rest.




20. During the first half-period of the motion, the graph that best represents the centripetal acceleration of the mass as a function of time is

(A) A (B) B (C) C (D) D (E) E

21. During the first half-period of the motion, the graph that best represents the magnitude of the tangential acceleration of the mass as a function of time is

(A) A (B) B (C) C (D) D (E) E

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22. The work done by the tension force as the object moves to its lowest point is

(A) $mgL(1 - \cos\theta)$ (B) $mgL(1 - \sin\theta)$ (C) $mgL\cos\theta$ (D) $mgL\sin\theta$ (E) 0

23. A 2 kg mass moves in one dimension under the influence of a position-dependent force given by $F(x) = 2x - 8x^3$. The work done by this force as the object moves from $x = 2$ m to $x = 1$ m is closest to

(A) −6 J (B) −27 J (C) 6 J (D) 27 J (E) 8 J

24. A mass $m$ falls from a height $H$ onto a previously compressed spring. The mass rebounds with twice its incoming speed. If the spring and mass are in contact for a time $T$, the average power delivered to the mass by the spring is

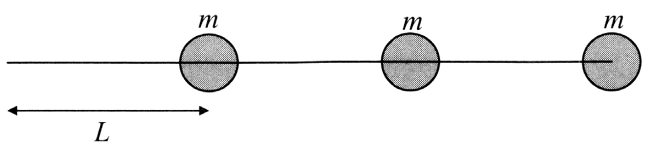
(A) $\frac{mgh}{T}$ (B) $\frac{2mgH}{T}$ (C) $\frac{\sqrt{2}mgH}{T}$ (D) $\frac{4mgH}{T}$ (E) $\frac{3mgH}{T}$

25. A mass $m$ moving with speed $v$ in the $+x$ direction collides head-on with a mass $4m$ moving with speed $\frac{1}{2}v$ in the $-x$ direction. If the two masses stick together, the velocity after collision is

(A) 0 (B) $\frac{1}{5}v$, $+x$ direction (C) $\frac{3}{5}v$, $+x$ direction
(D) $\frac{1}{5}v$, $-x$ direction (E) $\frac{3}{5}v$, $-x$ direction

26. A wheel with rotational inertia $I$ is rotating with a constant angular velocity $\omega_0$ when a torque is applied, slowing the wheel to $\frac{1}{2}\omega_0$ in a time $T$. The magnitude of the torque applied is

(A) $\frac{I\omega_0}{2T}$ (B) $\frac{I\omega_0}{T}$ (C) $\frac{3I\omega_0}{2T}$ (D) $\frac{2I\omega_0}{T}$ (E) $\frac{5I\omega_0}{2T}$




27. Three equal masses are connected at equal distances along a rod of length $3L$ and negligible mass. The moment of inertia about the left end of the rod is

(A) $3mL^2$ (B) $9mL^2$ (C) $14mL^2$ (D) $17mL^2$ (E) $27mL^2$

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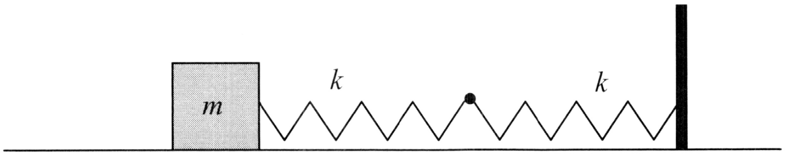
28. Planet X has an acceleration due to gravity at its surface that is 3 times that at Earth's surface. If the mass of X is twice the mass of the Earth, the radius of X in terms of $R_e$, the radius of Earth, is

(A) $\frac{2}{3}R_e$ (B) $\frac{3}{2}R_e$ (C) $\sqrt{\frac{2}{3}}R_e$ (D) $\sqrt{\frac{3}{2}}R_e$ (E) $\frac{2}{\sqrt{3}}R_e$




29. Two identical springs with spring constant $k$ are connected as shown in the figure, and a mass $m$ is attached. When the mass is displaced from equilibrium, the period of oscillation is

(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m}{2k}}$ (C) $2\pi\sqrt{\frac{2m}{k}}$ (D) $4\pi\sqrt{\frac{m}{k}}$ (E) $2\sqrt{2}\pi\sqrt{\frac{m}{k}}$

30. The position of a 3-kg mass moving in one dimension is given in meters as a function of time by the following equation: $x(t) = t^3 - 4t^2 + 7$. At $t = 1$ s, the force on the object is

(A) 12 N (B) 15 N (C) 36 N (D) −6 N (E) −2 N

31. A toy air gun can fire a dart to a height $H$ when fired straight up. What is the maximum height that the projectile will reach if it's fired at an angle of 60° with the vertical?

(A) $\frac{\sqrt{3}}{2}H$ (B) $\frac{3}{4}H$ (C) $\frac{1}{2}H$ (D) $\frac{1}{4}H$ (E) $\frac{1}{3}H$

32. A 40-kg child and an 80-kg man are ice skating in tandem, with the child in front of the man. When they're moving at 10 m/s, the man pushes the child so that she moves at 15 m/s relative to the ice. After she separates, what will be the velocity of their center of mass?

(A) 7.5 m/s (B) 9 m/s (C) 10 m/s (D) 12.5 m/s (E) 15 m/s

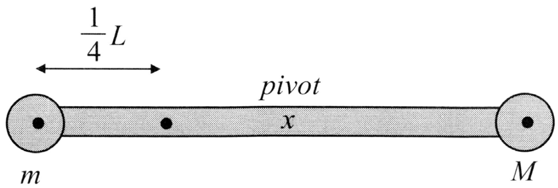
33. A circular hoop of mass $M$ and radius $R$ rolls down an incline without slipping, from an initial height $H$. The translational kinetic energy at the bottom of the incline is

(A) $mgH$ (B) $\frac{1}{2}mgH$ (C) $\frac{1}{3}mgH$ (D) $\frac{2}{3}mgH$ (E) $\frac{1}{4}mgH$

GO ON TO THE NEXT PAGE

34. A block slides down a rough incline, experiencing a constant force of friction. At the bottom of the incline, it encounters an uncompressed, massless spring and eventually rebounds back up the incline. After the mass has reached its highest point on the rebound, which of the following is true for the work done by gravity and the work done by the spring force?

| | Gravity | Spring |
|---|---|---|
| (A) | positive | 0 |
| (B) | negative | 0 |
| (C) | 0 | positive |
| (D) | negative | positive |
| (E) | 0 | 0 |




35. A stick of mass $m$ and length $L$ has its mass distributed nonuniformly so that its center of mass is $\frac{1}{4}L$ from the left. A mass $m$ is attached to the left end of the stick. What mass $M$ must be attached to the right end of the stick so that the system won't rotate when it's pivoted about the geometrical center of the stick?

(A) $\frac{1}{2}m$ (B) $m$ (C) $\frac{3}{2}m$ (D) $2m$ (E) $\frac{5}{2}m$

STOP
END OF SECTION I, MECHANICS

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION I, MECHANICS ONLY.

DO NOT TURN TO ANY OTHER TEST MATERIALS.

STOP

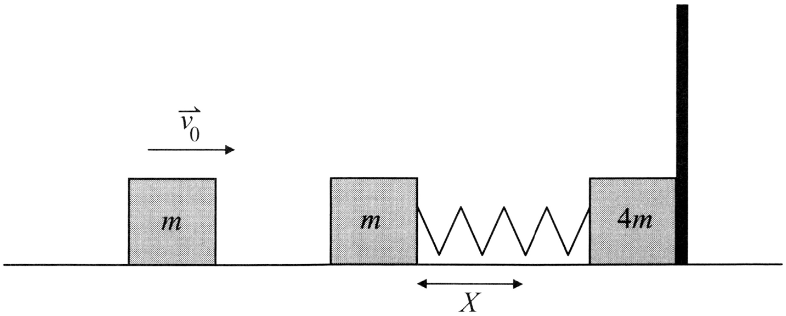
# AP PHYSICS C TEST

## SECTION II–MECHANICS

Time: 45 minutes

3 Questions

**Directions:** Answer all 3 questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight.




1. Two masses $m$ and $4m$ are connected by a spring on a smooth horizontal surface. The larger mass is currently in contact with a barrier that will not move, and a third mass $m$ moving with speed $v_0$ slides across the surface and strikes the smaller mass as shown in the figure. Upon impact, the two equal masses stick together, and the spring compresses a maximum $X$. Answer the following in terms of $m$, $v_0$, and $X$.

   (a) Determine the speed of the smaller masses immediately after impact.
   (b) Determine the force constant of the spring.
   (c) How much mechanical energy is lost as a result of the collision?

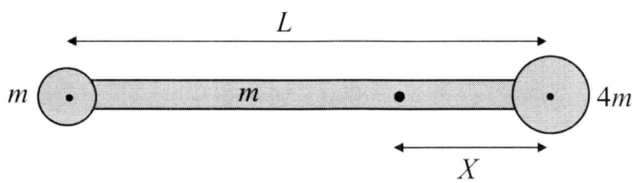
   The barrier is now removed so that the larger mass is free to move on the surface, and the collision is repeated. The same initial conditions are reproduced so that one of the smaller masses moving with speed $v_0$ collides with the other, sticking to it.

   (d) When the spring is at maximum compression, what will be the speed of each mass?
   (e) Determine the new maximum compression of the spring.

GO ON TO THE NEXT PAGE

2. A tall glass cylinder contains a clear liquid. When a small spherical mass m is gently placed into the liquid, it experiences a resistive force $\vec{F} = -bm\vec{v}$ proportional to the velocity of the mass as it falls. $b$ is a constant that depends on properties of the liquid.
   (a) What is the initial acceleration of the mass just as it is released?
   (b) Determine the terminal speed of the mass under the conditions described.
   (c) Write the differential equation that could be used to find the velocity of the mass at any time $t > 0$.
   (d) Determine the time it takes for the mass to reach $\frac{1}{2}$ its terminal speed.
   (e) You are given a supply of several different clear liquids. Describe an experimental procedure for determining the $b$-values for these liquids. You may use the apparatus described earlier and any of the following.

| | | |
|---|---|---|
| meter stick | stopwatch | set of known masses |
| roll of string | spring | |




3. Two masses $m$ and $4m$ are connected by light pins to the ends of a stick of mass $m$ and length $L$. The stick is pivoted to rotate vertically about a horizontal axis at the center of mass of the system, located a distance $X$ from the larger mass.
   (a) Determine the value of $X$ in terms of $m$ and $L$.
   (b) What is the moment of inertia of the system about its center of mass?

   With the system initially at rest and the stick horizontal, another mass $m$ is dropped onto the smaller mass. Just before it strikes the mass, it is moving *down* with speed $v$, and just after the collision, it is moving *up* with speed $\frac{v}{3}$.
   (c) Describe the motion of the pivoted system after the collision.
   (d) Find the angular speed of the stick after the collision.
   (e) Find the force exerted on the larger mass by its connecting pin when this mass is at its lowest position.

STOP
END OF SECTION II, MECHANICS

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, MECHANICS ONLY.

DO NOT TURN TO ANY OTHER TEST MATERIALS.

STOP

# 5 STEPS TO A 5 Practice Exam

## Physics C—Mechanics Practice Exam 1—Multiple-Choice Questions

**Time:** 45 minutes. You may refer to the constants sheet and the equation sheet, both of which are found in the appendix. You may use a calculator.

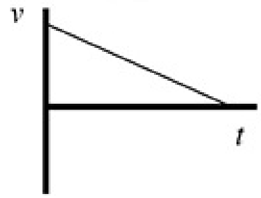
**1.** A cannon is mounted on a truck that moves forward at a speed of 5 m/s. The operator wants to launch a ball from a cannon so the ball goes as far as possible before hitting the level surface. The muzzle velocity of the cannon is 50 m/s. At what angle from the horizontal should the operator point the cannon?

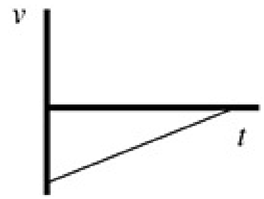
(A) 5°
(B) 41°
(C) 45°
(D) 49°
(E) 85°

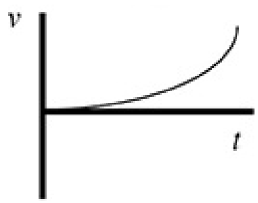
**2.** A car moving with speed $v$ reaches the foot of an incline of angle $\theta$. The car coasts up the incline without using the engine. Neglecting friction and air resistance, which of the following is correct about the magnitude of the car's horizontal acceleration $a_x$ and vertical acceleration $a_y$?

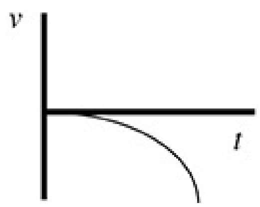
(A) $a_x = 0$; $a_y < g$
(B) $a_x = 0$; $a_y = g$
(C) $a_x < g$; $a_y < g$
(D) $a_x < g$; $a_y = g$
(E) $a_x < g$; $a_y > g$

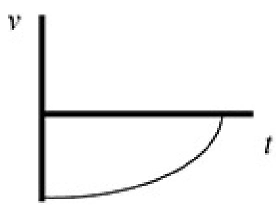
**3.** A bicycle slows down with an acceleration whose magnitude increases linearly with time. Which of the following velocity–time graphs could represent the motion of the bicycle?

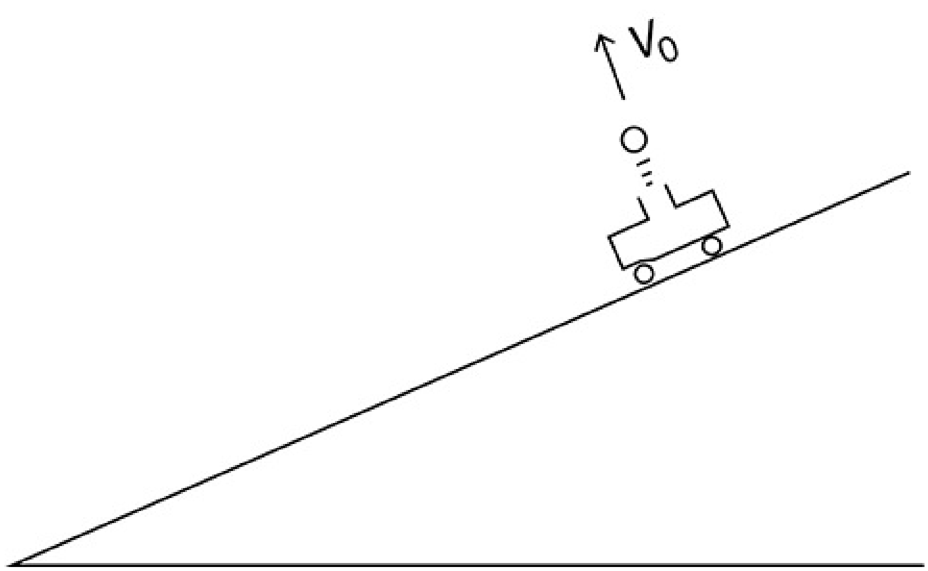
(A) 


(B) 


(C) 


(D) 


(E) 


**4.** A cart is sliding down a low friction incline. A device on the cart launches a

 [108]

ball, forcing the ball perpendicular to the incline, as shown above. Air resistance is negligible. Where will the ball land relative to the cart, and why?

(A) The ball will land in front of the cart, because the ball's acceleration component parallel to the plane is greater than the cart's acceleration component parallel to the plane.
(B) The ball will land in front of the cart, because the ball has a greater magnitude of acceleration than the cart.
(C) The ball will land in the cart, because both the ball and the cart have the same component of acceleration parallel to the plane.
(D) The ball will land in the cart, because both the ball and the cart have the same magnitude of acceleration.
(E) The ball will land behind the cart, because the ball slows down in the horizontal direction after it leaves the cart.

**5.** The quantity "jerk," *j*, is defined as the time derivative of an object's acceleration,

$$j = \frac{da}{dt} = \frac{d^3x}{dt^3}.$$

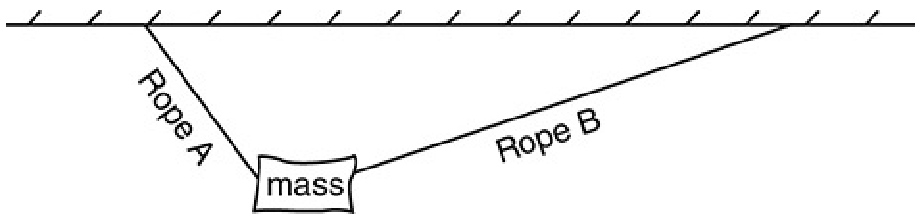
What is the physical meaning of the area under a graph of jerk vs. time?

(A) The area represents the object's acceleration.
(B) The area represents the object's change in acceleration.
(C) The area represents the object's change in velocity.
(D) The area represents the object's velocity.
(E) The area represents the object's change in position.

**6.** A particle moves along the $x$-axis with a position given by the equation $x(t) = 5 + 3t$, where $x$ is in meters, and $t$ is in seconds. The positive direction is east. Which of the following statements about the particle is FALSE.

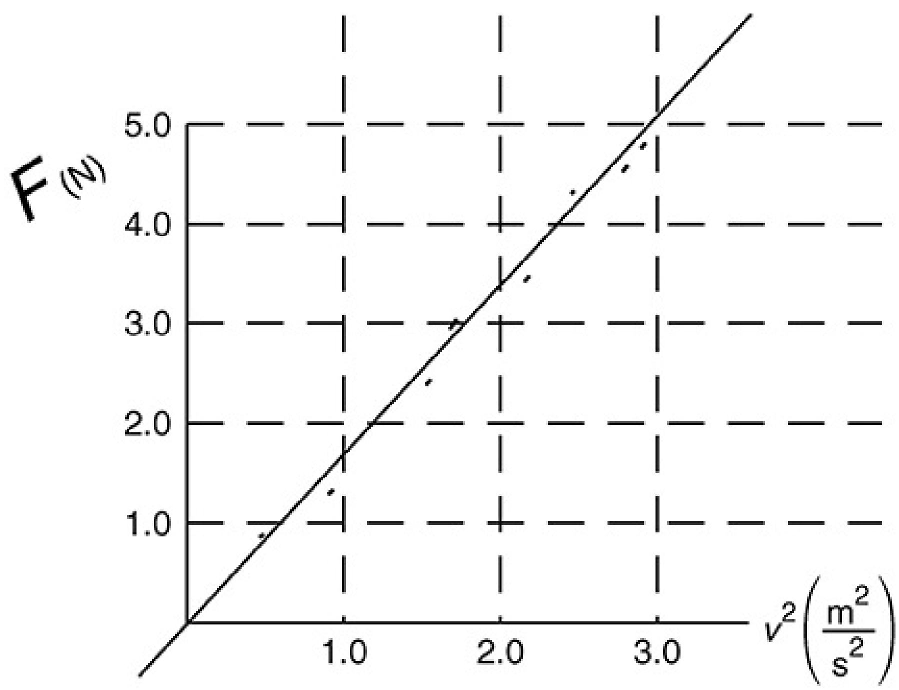
(A) The particle is east of the origin at $t = 0$.
(B) The particle is at rest at $t = 0$.
(C) The particle's velocity is constant.
(D) The particle's acceleration is constant.
(E) The particle will never be west of position $x = 0$.


[109]




**7.** A mass hangs from two ropes at unequal angles, as shown above. Which of the following makes correct comparisons of the horizontal and vertical components of the tension in each rope?

| | Horizontal Tension | Vertical Tension |
|---|---|---|
| (A) | greater in rope *B* | greater in rope *B* |
| (B) | equal in both ropes | greater in rope *A* |
| (C) | greater in rope *A* | greater in rope *A* |
| (D) | equal in both ropes | equal in both ropes |
| (E) | greater in rope *B* | equal in both ropes |




**8.** The force of air resistance *F* on a mass is found to obey the equation *F* =

$bv^2$, where $v$ is the speed of the mass, for the range of speeds investigated in an experiment. A graph of $F$ vs. $v^2$ is shown above. What is the value of $b$?

(A) 0.83 kg/m
(B) 1.7 kg/m
(C) 3.0 kg/m
(D) 5.0 kg/m
(E) 1.0 kg/m

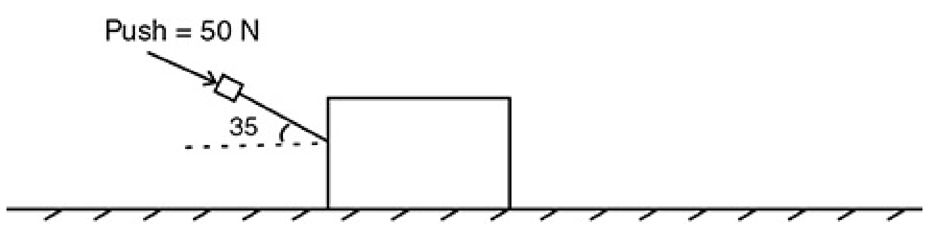
**9.** A box sits on an inclined plane without sliding. As the angle of the plane (measured from the horizontal) increases, the normal force

(A) increases linearly
(B) decreases linearly
(C) does not change
(D) decreases nonlinearly
(E) increases nonlinearly

**10.** Which of the following conditions are necessary for an object to be in static equilibrium?

I. The vector sum of all torques on the object must equal zero.
II. The vector sum of all forces on the object must equal zero.
III. The sum of the object's potential and kinetic energies must be zero.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

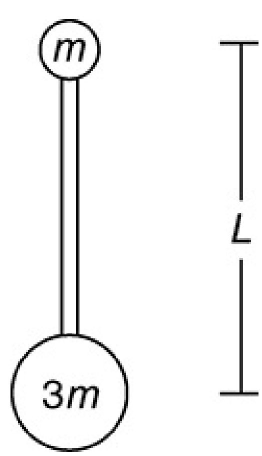



**11.** A student pushes a big 16-kg box across the floor at constant speed. He pushes with a force of 50 N angled 35° from the horizontal, as shown in the diagram above. If the student pulls rather than pushes the box at the same angle, while maintaining a constant speed, what will happen to the force of friction?

(A) It must increase.
(B) It must decrease.
(C) It must remain the same.
(D) It will increase only if the speed is greater than 3.1 m/s.
(E) It will increase only if the speed is less than 3.1 m/s.

**12.** Consider a system consisting only of the Earth and a bowling ball, which moves upward in a parabola above Earth's surface. The downward force of Earth's gravity on the ball, and the upward force of the ball's gravity on the Earth, form a Newton's third law force pair. Which of the following statements about the ball is correct?

(A) The ball must be in equilibrium since the upward forces must cancel downward forces.
(B) The ball accelerates toward the Earth because the force of gravity on the ball is greater than the force of the ball on the Earth.
(C) The ball accelerates toward the Earth because the force of gravity on the ball is the only force acting on the ball.
(D) The ball accelerates away from Earth because the force causing the ball to move upward is greater than the force of gravity on the ball.
(E) The ball accelerates away from Earth because the force causing the ball to move upward plus the force of the ball on the Earth are together greater than the force of gravity on the ball.




**13.** A mass $m$ is attached to a mass $3m$ by a rigid bar of negligible mass and length $L$. Initially, the smaller mass is located directly above the larger mass, as shown above. How much work is necessary to flip the rod 180° so that the larger mass is directly above the smaller mass?

(A) $4mgL$
(B) $2mgL$
(C) $mgL$
(D) $4\pi mgL$
(E) $2\pi mgL$

**14.** A ball rolls horizontally with speed $v$ off of a table a height $h$ above the ground. Just before the ball hits the ground, what is its speed?

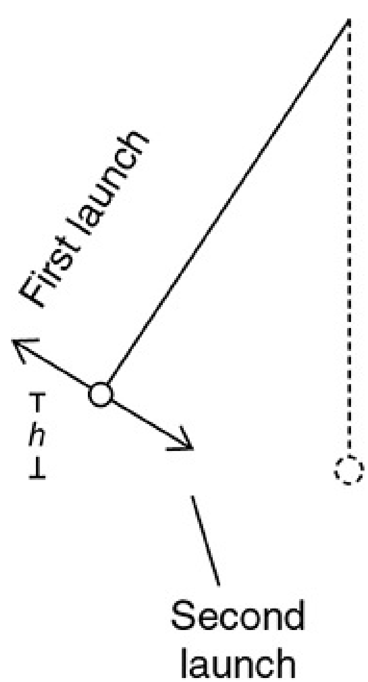
(A) $\sqrt{2gh}$

(B) $v\sqrt{2gh}$

(C) $\sqrt{v^2 + 2gh}$

(D) $v$

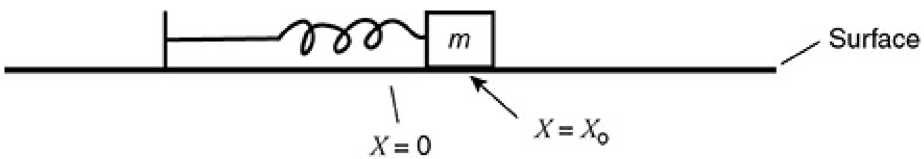
(E) $v + \sqrt{2gh}$




 [113]

**15.** A pendulum is launched into simple harmonic motion in two different ways, as shown above, from a point that is a height *h* above its lowest point. During both launches, the bob is given an initial speed of 3.0 m/s. On the first launch, the initial velocity of the bob is directed upward along the pendulum's path, and on the second launch it is directed downward along the pendulum's path. Which launch will cause the pendulum to swing with the larger amplitude?

(A) the first launch
(B) the second launch
(C) Both launches produce the same amplitude.
(D) The answer depends on the initial height *h*.
(E) The answer depends on the length of the supporting rope.




**16.** The mass *M* is moving to the right with velocity $v_0$ at position $x = x_0$. Neglect friction. The spring has force constant *k*. What is the total mechanical energy of the block at this position?

(A) $\frac{1}{2}mv_0^2$

(B) $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$

(C) $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 + mgx_0$

(D) $mgx_0 + \frac{1}{2}mv_0^2$

(E) $mgx_0 + \frac{1}{2}kx_0^2$

**17.** A sphere, a cube, and a cylinder, all of equal mass, are released from rest from the top of a short incline. The surface of the incline is extremely slick, so much so that the objects do not rotate when released, but rather slide with negligible friction. Which reaches the base of the incline first?

(A) the sphere
(B) the cube
(C) the cylinder


[114]

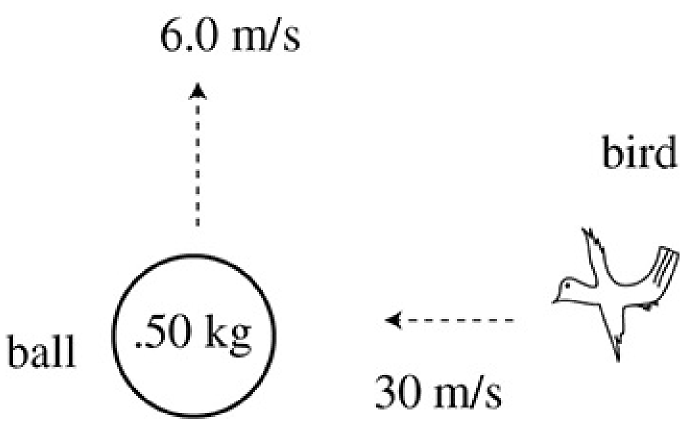
(D) All reach the base at the same time.
(E) The answer depends on the relative sizes of the objects.


**18.** Block *B* is at rest on a smooth tabletop. It is attached to a long spring, which is in turn anchored to the wall. Block *A* slides toward and collides with block *B*. Consider two possible collisions:

Collision I: Block *A* bounces back off of block *B*.
Collision II: Block *A* sticks to block *B*.

Which of the following is correct about the speed of block *B* immediately after the collision?

(A) It is faster in case II than in case I ONLY if block *B* is heavier.
(B) It is faster in case I than in case II ONLY if block *B* is heavier.
(C) It is faster in case II than in case I regardless of the mass of each block.
(D) It is faster in case I than in case II regardless of the mass of each block.
(E) It is the same in either case regardless of the mass of each block.

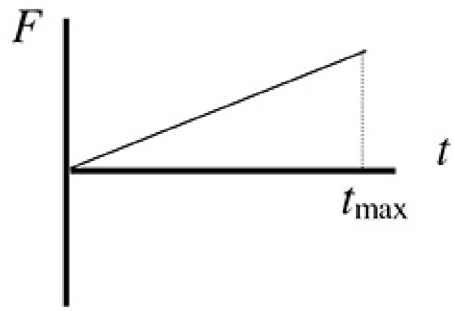



**19.** A 0.30-kg bird is flying from right to left at 30 m/s. The bird collides with and sticks to a 0.50-kg ball that is moving straight up with speed 6.0 m/s. What is the magnitude of the momentum of the ball/bird combination immediately after collision?

(A) 12.0 N·s
(B) 9.5 N·s
(C) 9.0 N·s


[115]

(D) 6.0 N·s
(E) 3.0 N·s




**20.** The force $F$ on a mass is shown above as a function of time $t$. Which of the following methods can be used to determine the impulse experienced by the mass?

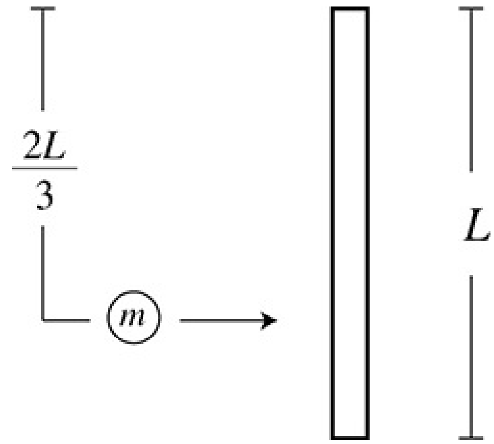
I. multiplying the average force by $t_{max}$
II. calculating the area under the line on the graph
III. taking the integral $\int_0^{t_{max}} F \cdot dt$

(A) II only
(B) III only
(C) II and III only
(D) I and II only
(E) I, II, and III

**21.** A projectile is launched on level ground in a parabolic path so that its range would normally be 500 m. When the projectile is at the peak of its flight, the projectile breaks into two pieces of equal mass. One of these pieces falls straight down, with no further horizontal motion. How far away from the launch point does the other piece land?

(A) 250 m
(B) 375 m
(C) 500 m
(D) 750 m
(E) 1000 m

Questions 22 and 23




A rigid rod of length $L$ and mass $M$ is floating at rest in space far from a gravitational field. A small blob of putty of mass $m < M$ is moving to the right, as shown above. The putty hits and sticks to the rod a distance $2L/3$ from the top end.

**22.** How will the rod/putty contraption move after the collision?

(A) The contraption will have no translational motion, but will rotate about the rod's center of mass.
(B) The contraption will have no translational motion, but will rotate about the center of mass of the rod and putty combined.
(C) The contraption will move to the right and rotate about the position of the putty.
(D) The contraption will move to the right and rotate about the center of mass of the rod and putty combined.
(E) The contraption will move to the right and rotate about the rod's center of mass.

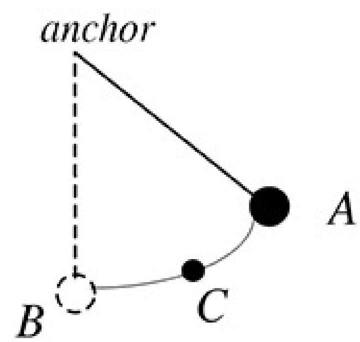
**23.** What quantities are conserved in this collision?

(A) linear and angular momentum, but not kinetic energy
(B) linear momentum only
(C) angular momentum only
(D) linear and angular momentum, and linear but not rotational kinetic energy
(E) linear and angular momentum, and linear and rotational kinetic energy


[117]

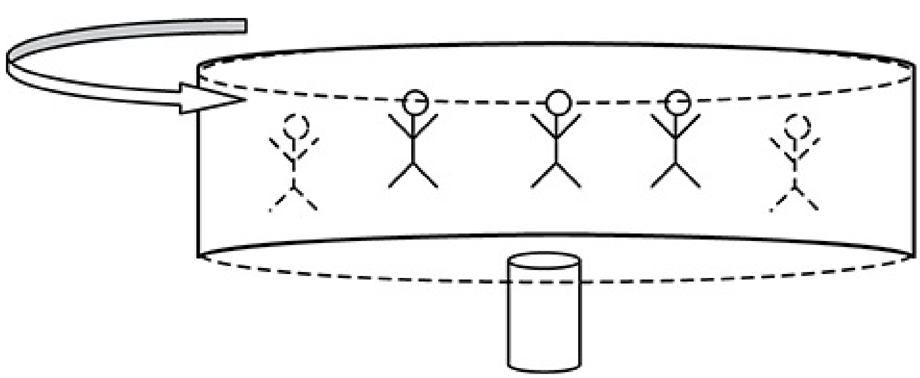
**24.** A car rounds a banked curve of uniform radius. Three forces act on the car: a friction force between the tires and the road, the normal force from the road, and the weight of the car. Which provides the centripetal force which keeps the car in circular motion?

(A) the friction force alone
(B) the normal force alone
(C) the weight alone
(D) a combination of the normal force and the friction force
(E) a combination of the friction force and the weight




**25.** A ball of mass *m* anchored to a string swings back and forth to a maximum position *A*, as shown above. Point *C* is partway back to the vertical position. What is the direction of the mass's acceleration at point *C*?

(A) along the mass's path toward point *B*
(B) toward the anchor
(C) away from the anchor
(D) between a line toward the anchor and a line along the mass's path
(E) along the mass's path toward point *A*




[118]

**26.** In a carnival ride, people of mass *m* are whirled in a horizontal circle by a floorless cylindrical room of radius *r*, as shown in the diagram above. If the coefficient of friction between the people and the tube surface is $\mu$, what minimum speed is necessary to keep the people from sliding down the walls?

(A) $\sqrt{\mu r g}$

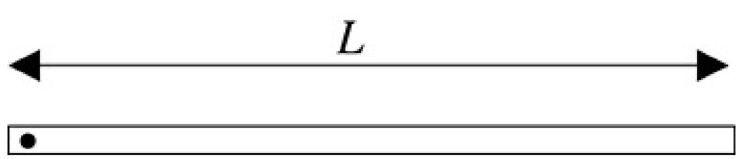
(B) $\sqrt{\frac{rg}{\mu}}$

(C) $\sqrt{\frac{\mu}{rg}}$

(D) $\sqrt{\frac{1}{\mu r g}}$

(E) $\sqrt{\mu m g}$

<u>Questions 27 and 28</u>




The uniform, rigid rod of mass *m*, length *L*, and rotational inertia *I* shown above is pivoted at its left-hand end. The rod is released from rest from a horizontal position.

**27.** What is the linear acceleration of the rod's center of mass the moment after the rod is released?

(A) $\frac{mgL^2}{2I}$

(B) $\frac{mgL^2}{4I}$

(C) $\frac{mgL^2}{I}$

(D) $\frac{mgL}{2I}$

(E) $\frac{2mgL^2}{I}$

**28.** What is the linear speed of the rod's center of mass when the mass passes through a vertical position?

(A) $\sqrt{\frac{mgL^3}{8I}}$

(B) $\sqrt{\frac{mg\pi L^3}{4I}}$

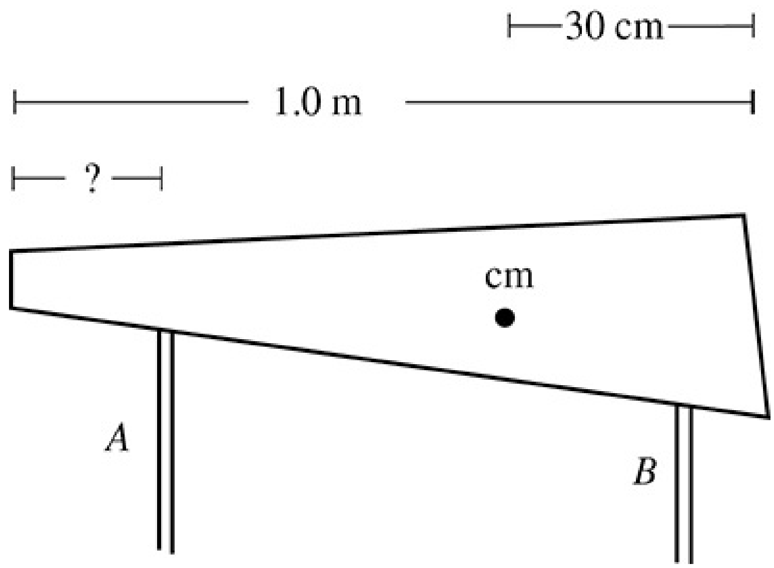
(C) $\sqrt{\frac{mg\pi L^3}{8I}}$

(D) $\sqrt{\frac{mgL^3}{4I}}$

(E) $\sqrt{\frac{mgL^3}{2I}}$



[120]




**29.** The 1.0-m-long nonuniform plank, shown above, has weight 1000 N. It is to be supported by two rods, *A* and *B*, as shown above. The center of mass of the plank is 30 cm from the right edge. Each support bears half the weight of the plank. If support *B* is 10 cm from the right-hand edge, how far from the left-hand edge should support *A* be?

(A) 0 cm
(B) 10 cm
(C) 30 cm
(D) 50 cm
(E) 70 cm

**30.** A mass *m* on a spring oscillates on a horizontal surface with period *T*. The total mechanical energy contained in this oscillation is *E*. Imagine that instead a new mass 4*m* oscillates on the same spring with the same amplitude. What is the new period and total mechanical energy?

| | Period | Total Mechanical Energy |
|---|---|---|
| (A) | $T$ | $E$ |
| (B) | $2T$ | $E$ |
| (C) | $2T$ | $2E$ |
| (D) | $T$ | $4E$ |
| (E) | $2T$ | $16E$ |

**31.** A mass *m* is attached to a horizontal spring of spring constant *k*. The spring oscillates in simple harmonic motion with amplitude *A*. What is the maximum speed of this simple harmonic oscillator?

(A) $2\pi\sqrt{\frac{m}{k}}$

(B) $2\pi A\sqrt{\frac{m}{k}}$

(C) $2\pi A\sqrt{\frac{k}{m}}$

(D) $A\sqrt{\frac{k}{m}}$

(E) $A\sqrt{\frac{m}{k}}$

**32.** An empty bottle goes up and down on the surface of the ocean, obeying the position function $x = A\cos(\omega t)$. How much time does this bottle take to travel once from its lowest position to its highest position?

(A) $\frac{2\pi}{\omega}$

(B) $\frac{\pi}{\omega}$

(C) $\frac{4\pi}{\omega}$

(D) $\frac{\pi}{2\omega}$

(E) $\dfrac{\pi}{4\omega}$

**33.** The Space Shuttle orbits 300 km above the Earth's surface; the Earth's radius is 6400 km. What is the acceleration due to Earth's gravity experienced by the Space Shuttle?

(A) 4.9 $m/s^2$
(B) 8.9 $m/s^2$
(C) 9.8 $m/s^2$
(D) 10.8 $m/s^2$
(E) zero

**34.** An artificial satellite orbits Earth just above the atmosphere in a circle with constant speed. A small meteor collides with the satellite at point *P* in its orbit, increasing its speed by 1%, but not changing the instantaneous direction of the satellite's velocity. Which of the following describes the satellite's new orbit?

(A) The satellite now orbits in an ellipse, with *P* as the farthest approach to Earth.
(B) The satellite now orbits in an ellipse, with *P* as the closest approach to Earth.
(C) The satellite now orbits in a circle of larger radius.
(D) The satellite now orbits in a circle of smaller radius.
(E) The satellite cannot maintain an orbit, so it flies off into space.

**35.** Mercury orbits the sun in about one-fifth of an Earth year. If 1 AU is defined as the distance from the Earth to the sun, what is the approximate distance between Mercury and the sun?

(A) (1/25) AU
(B) (1/9) AU
(C) (1/5) AU
(D) (1/3) AU
(E) (1/2) AU

## STOP. End of Physics C—Mechanics Practice Exam 1—Multiple-Choice Questions

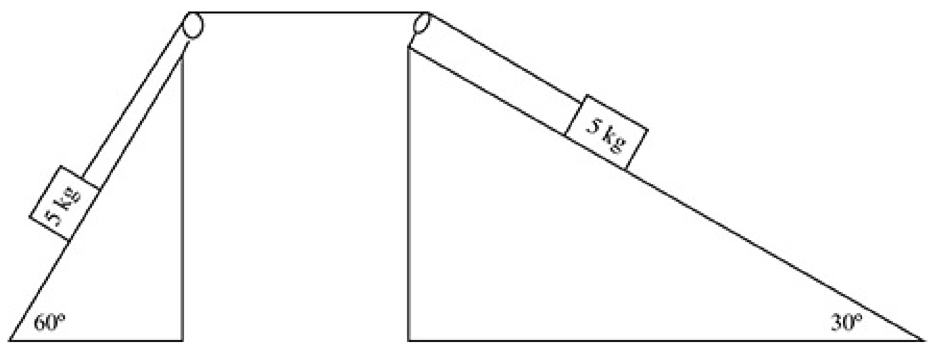


[124]

# Physics C—Mechanics Practice Exam 1—Free-Response Questions

**Time:** 45 minutes. You may refer to the constants sheet and the equation, both of which are found in the appendix. You may use a calculator.

## CM 1




Two 5-kg masses are connected by a light string over two massless, frictionless pulleys. Each block sits on a frictionless inclined plane, as shown above. The blocks are released from rest.

(a) Determine the magnitude of the acceleration of the blocks.

Now assume that the 30° incline provides a resistive force which depends on speed *v*. This resistive force causes the entire system's acceleration to be given by the expression

$$a = 1.8 - 0.03v$$

where a speed *v* in m/s gives an acceleration in m/s$^2$. The blocks are again released from rest.

(b) i. On the axes below, sketch a graph of the speed of the 5 kg block as a function of time. Label important values, including any asymptotes and intercepts.

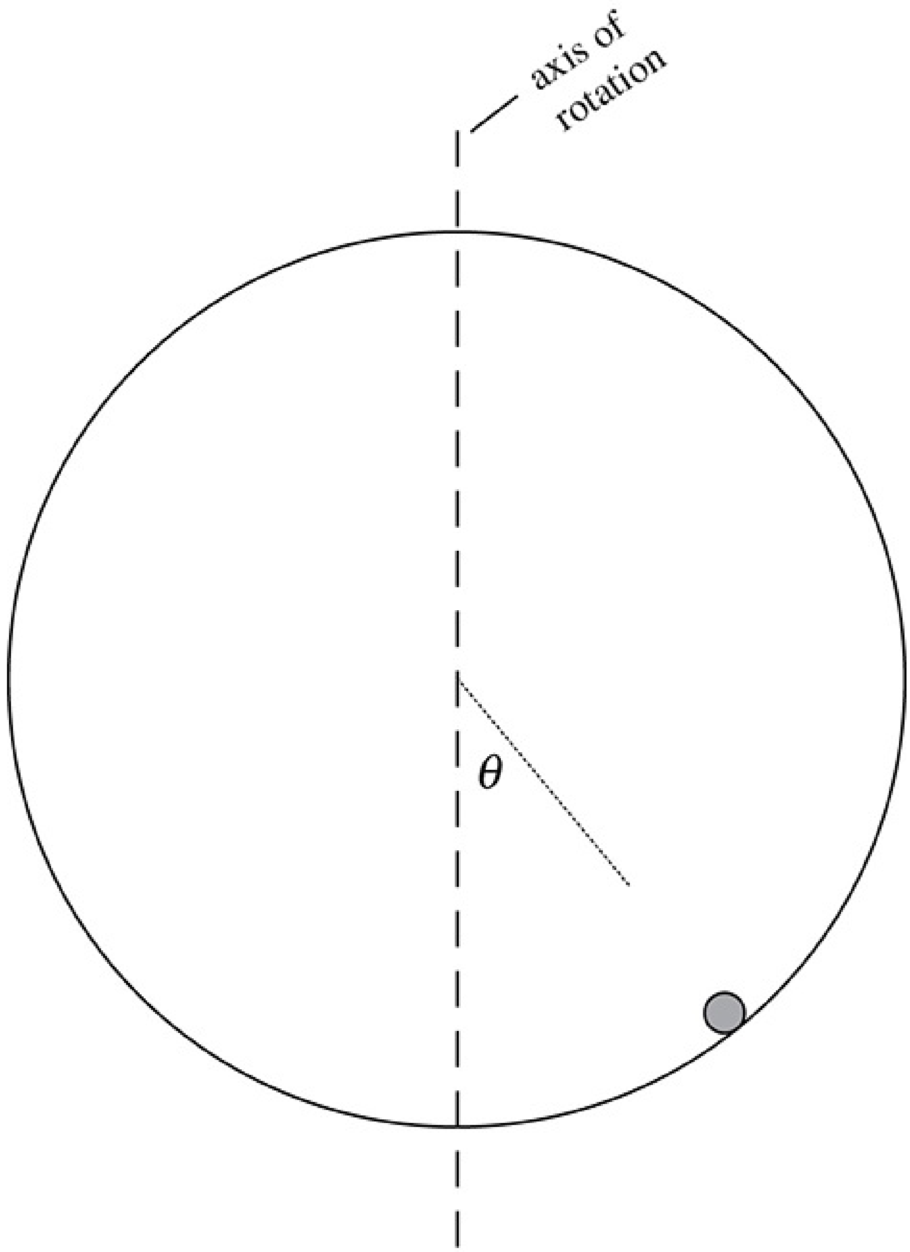

ii. Explain how the expression for acceleration leads to the graph you drew.

(c) Explain how to figure out the terminal speed of the 5 kg block.

(d) The terminal speed is 60 m/s – that's a typical speed in automobile racing. Explain briefly why this result is physically reasonable, even though the blocks are on a track in a physics laboratory.

## CM 2


[126]




A hollow glass sphere of radius 8.0 cm rotates about a vertical diameter with frequency 5 revolutions per second. A small wooden ball of mass 2.0 g rotates inside the sphere, as shown in the diagram above.

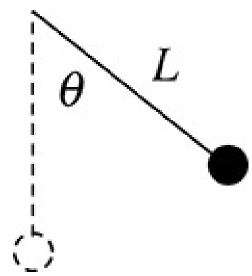
(a) Draw a free-body diagram indicating the forces acting on the wooden ball when it is at the position shown in the picture above.

(b) Calculate the angle $\theta$, shown in the diagram above, to which the ball rises.

(c) Calculate the linear speed of the wooden ball as it rotates.

(d) The wooden ball is replaced with a steel ball of mass 20 g. Describe how the angle $\theta$ to which the ball rises will be affected. Justify your answer.

## CM 3



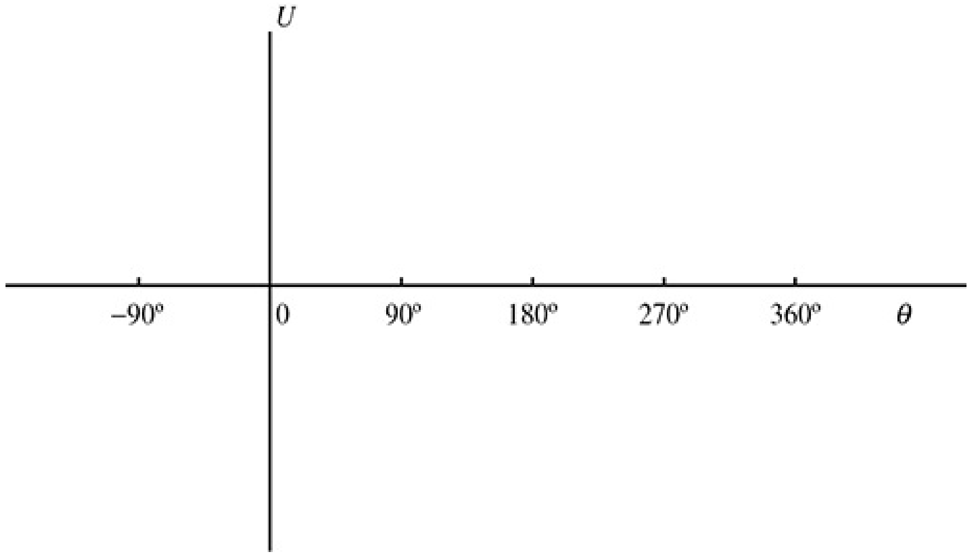

A heavy ball of mass $m$ is attached to a light but rigid rod of length $L$. The rod is pivoted at the top and is free to rotate in a circle in the plane of the page, as shown above.

(a) The mass oscillates to a maximum angle $\theta$. On the picture of the mass $m$ below, draw a vector representing the direction of the NET force on the mass while it is at angle $\theta$. Justify your choice of direction.

(b) Is the magnitude of the net force at the maximum displacement equal to $mg \sin\theta$ or $mg \cos\theta$? Choose one and justify your choice.

(c) Derive an expression for the ball's potential energy $U$ as a function of the angle $\theta$. Assume that a negative angle represents displacement from the vertical in the clockwise direction.

(d) On the axes below, sketch a graph of the mass's potential energy $U$ as a function of the angle $\theta$ for angles between −90° and +360°. Label maximum and minimum values on the vertical axis.




(e) The pendulum is considered a classic example of simple harmonic motion when it undergoes small-amplitude oscillation. With specific reference to the graph you made in part (d), explain why the assumption of simple harmonic motion is valid.

**STOP. End of Physics C—Mechanics Practice Exam 1—Free-Response Questions**

 [129]

# Barron's Practice Exam

# Practice Test 1

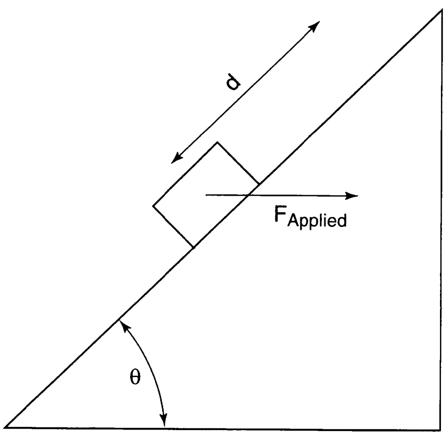
## MULTIPLE-CHOICE QUESTIONS

**Directions:** Each multiple-choice question is followed by five answer choices. For each question, choose the best answer and fill in the corresponding circle on the answer sheet. You may refer to the formula sheet in the Appendix (pages 639–642).

### Mechanics

1. A constant horizontal force $F$ applied to a block on an incline causes the block to move a distance $d$ along the incline, as shown. If the coefficient of friction between the block and the incline is $\mu_k$, what is the work done by the applied force? (Do not assume that the block moves at constant velocity.)

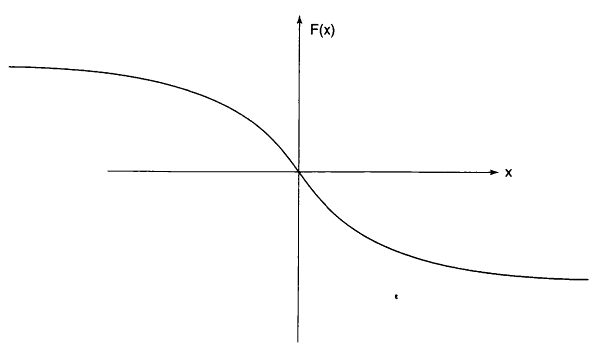
   (A) $W = Fd\sin\theta$
   (B) $W = Fd\cos\theta$
   (C) $W = mgd\sin\theta$
   (D) $W = mgd\sin\theta - \mu_k mgd\cos\theta$
   (E) $W = mgd\sin\theta + \mu_k d(mg\cos\theta + F\sin\theta)$




**Question 1**

2. The restoring force that acts on a mass is shown. For small amplitudes, the mass moves in simple harmonic motion with a period $T$. Which of the following is true of oscillations with larger amplitudes?

   (A) The motion is SHM with a period of $T$
   (B) The motion is SHM with a period smaller than $T$.
   (C) The motion is SHM with a period larger than $T$.
   (D) The motion is no longer SHM, and its period is smaller than $T$.
   (E) The motion is no longer SHM, and its period is larger than $T$.




**Question 2**

GO ON TO THE NEXT PAGE

PRACTICE TEST 1

3. A simple pendulum of length $\ell$ is moved to Mars (where $g = 3.71\ \text{m/s}^2$). Its period on Earth is 2.0 s. Its period on Mars is

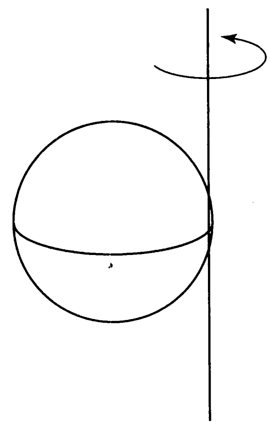
(A) 0.02 s
(B) 2.0 s
(C) 3.25 s
(D) 600 s
(E) 6,000 s

4. At the instant that a mass in simple harmonic motion passes through its equilibrium position its acceleration is

(A) maximum
(B) zero
(C) oppositely directed to its velocity
(D) parallel to its velocity
(E) cannot be determined without additional information

5. A solid sphere has a rotational inertia of $\frac{7}{5}Mr^2$ when rotated about an axis tangent to its surface as shown. What is the rotational inertia of the sphere when it is rotated about a line passing through the center of the sphere?

(A) $\frac{2}{5}Mr^2$
(B) $\frac{3}{5}Mr^2$
(C) $\frac{7}{5}Mr^2$
(D) $\frac{12}{5}Mr^2$
(E) $\frac{5}{14}Mr^2$



**Question 5**

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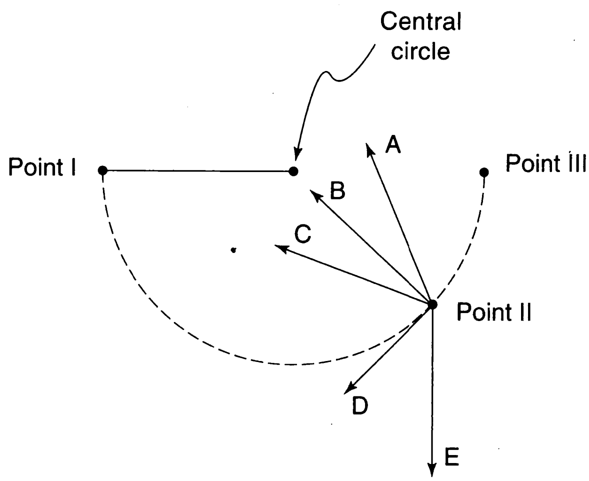
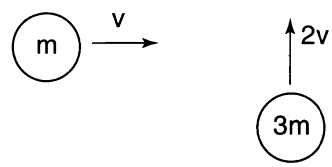
6. As shown in the figure, a pendulum is released with the string parallel to the ground at point I, resulting in its swinging through a 180° angle between points I and III. When the pendulum is at point II and moving downward, what is the direction of its acceleration?




**Question 6**

7. A mass $m$ moving to the right with speed $v$ collides and sticks to a mass of $3m$ moving upward with a speed $2v$ as shown. What is the final velocity of the two masses?

(A) $\frac{3}{2}v\hat{i} + \frac{1}{4}v\hat{j}$
(B) $\frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}$
(C) $\frac{1}{3}v\hat{i} + \frac{2}{3}v\hat{j}$
(D) $\frac{2}{3}v\hat{i} + \frac{1}{3}v\hat{j}$
(E) none of the above




**Question 7**

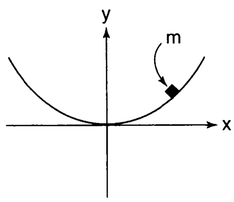
8. If a force of 2 N does work at the rate of $-2\sqrt{2}$ W on an object moving with a speed of 2 m/s, the angle between the force and the velocity vector must be

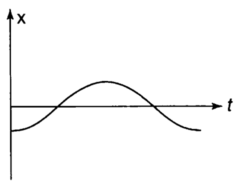
(A) 45°
(B) 120°
(C) 135°
(D) 150°
(E) It is impossible to deliver a negative power.

PRACTICE TEST 1

GO ON TO THE NEXT PAGE

9. A mass $m$ is released from rest at $t = 0$ in a parabolic depression as shown. If there is no friction between the mass and the surface, which of the graphs shows the mass's $x$-position as a function of time, assuming that the mass's displacement from the origin is small?




(A)
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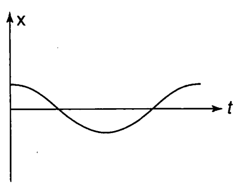


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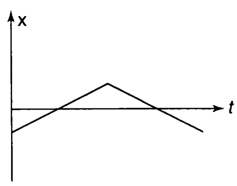


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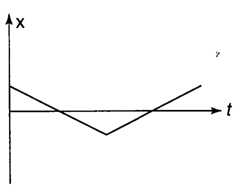
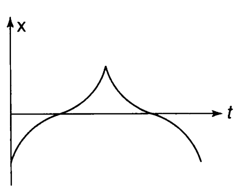



(E)



**Question 9**

10. Referring to the situation introduced in question 9, which of the graphs shows the $x$-component of the object's acceleration as a function of time?

*<u>Questions 11–13</u> refer to a projectile launched from Earth reaching a height equal to Earth's radius before returning to land. (Neglect air resistance.)*

11. When is the power exerted by the gravitational force the greatest?

   (A) at the instant after the projectile is launched
   (B) at the peak of the trajectory
   (C) at the instant before the projectile hits Earth
   (D) The power is maximized at both (A) and (B).
   (E) The power is constant.

12. At which of the following points is the acceleration the greatest in magnitude?

   (A) at the instant after the projectile is launched
   (B) at the peak of the trajectory
   (C) at the instant before the projectile hits Earth
   (D) The acceleration is maximized at both (A) and (C).
   (E) The acceleration is constant.

13. Compare the gravitational potential energy when the particle is launched to the potential energy when the particle is at the peak of its trajectory.

   (A) $U_{peak} = U_{launch}$ because of conservation of energy.
   (B) $|U_{peak}| > |U_{launch}|$ and $U_{peak} > 0$
   (C) $|U_{peak}| < |U_{launch}|$ and $U_{peak} < 0$
   (D) $|U_{peak}| < |U_{launch}|$ and $U_{peak} > 0$
   (E) $|U_{peak}| > |U_{launch}|$ and $U_{peak} < 0$

GO ON TO THE NEXT PAGE

14. A wheel accelerates at uniform angular acceleration $\alpha$ from rest. If its speed after one full rotation is $\omega$, what was its speed after half a rotation?

(A) $\frac{1}{2}\omega$
(B) $\omega$
(C) $\frac{3}{4}\omega$
(D) $(1/\sqrt{2})\omega$
(E) $\frac{1}{4}\omega$

15. A mass $m$ is suspended in an elevator. In terms of the tension $T$, of the string and the weight of the mass, what is the upward acceleration of the elevator?

(A) $T - mg$
(B) $mg - T$
(C) $(T/m) + g$
(D) $(T/m) - g$
(E) $g - T/m$

16. A mass initially at rest explodes into two fragments with masses $m_1$ and $m_2$. If $m_1$ subsequently moves with a speed $v_1$, how much kinetic energy was liberated in the explosion?

(A) $\frac{1}{2}m_1v_1^2$
(B) $\frac{1}{2}m_1v_1^2(1+m_1/m_2)$
(C) $\frac{1}{2}m_1v_1^2(1+m_2/m_1)$
(D) $\frac{1}{2}m_1v_1^2(1+m_2^2/m_1^2)$
(E) Not enough information is given.

17. A projectile is launched on level ground with an initial speed $v_0$ and angle of elevation $\theta$. All of the following statements concerning the projectile right before it lands are correct *except*

(A) The acceleration is equal to $g$ downward.
(B) The power exerted by the gravitational force is maximized.
(C) The time the projectile was in the air is equal to $2v_0\sin\theta/g$.
(D) The projectile has the same kinetic energy as when it was launched.
(E) The projectile has the same velocity as when it was launched.

18. For a system in simple harmonic motion, which of the following statements is true? (The equilibrium point refers to the point in the middle of the particle's oscillation, which could potentially be a static equilibrium point; the turning points are points where the particle reverses direction.)

(A) The acceleration is maximized at the equilibrium point.
(B) The velocity is maximized at the turning points.
(C) The restoring force is maximized at the equilibrium point.
(D) The displacement is maximized at the turning points.
(E) None of the above are true; it depends on the particular situation.

19. An object slides with horizontal velocity $v_0$ off a cliff of height $h$. How far from the base of the cliff does the object hit the ground?

(A) $v_0g/h$
(B) $v_0h/2g$
(C) $2gv_0/h$
(D) $v_0\sqrt{2h/g}$
(E) $v_0\sqrt{h/2g}$

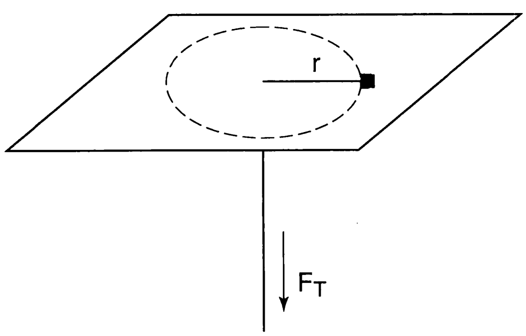
GO ON TO THE NEXT PAGE

20. The angular position of a wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$, where $\theta$ is measured in radians and $t$ in seconds. When is the torque on the wheel equal to zero?

(A) $t = 0.33$ s
(B) $t = 0.5$ s
(C) $t = 1$ s
(D) $t = 2$ s
(E) $t = 3$ s

21. A small mass attached to a string rotates on a frictionless tabletop as shown. If the tension in the string is increased, causing the radius of the circular motion to decrease by a factor of 2, what effect will this have on the kinetic energy of the mass?

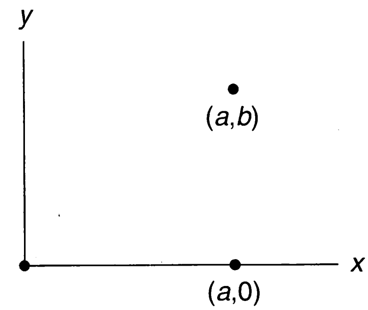
(A) It will decrease by a factor of 2.
(B) It will remain constant.
(C) It will increase by a factor of 2.
(D) It will increase by a factor of 4.
(E) It will increase by a factor of 8.




**Question 21**

22. Three point masses of equal mass are located as shown in the figure. What is the $x$-coordinate of the center of mass of this system?

(A) $\frac{a}{2}$
(B) $\frac{2a}{3}$
(C) $\frac{a}{3}$
(D) $\frac{b}{2}$
(E) $\frac{3a}{4}$




**Question 22**

GO ON TO THE NEXT PAGE

23. When two springs are attached in parallel to a mass $m$ as shown, what is the equivalent spring constant?

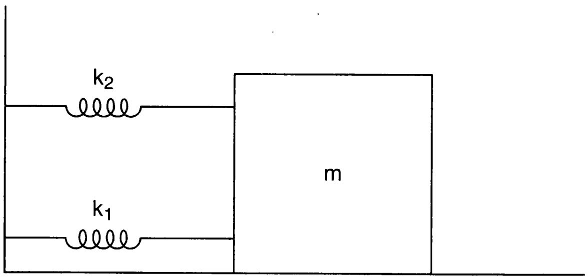
(A) $k_{eq} = k_1 + k_2$

(B) $k_{eq} = \dfrac{1}{1/k_1 + 1/k_2}$

(C) $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$

(D) $k_{eq} = \sqrt{k_1^2 + k_2^2}$

(E) none of the above




**Question 23**

24. A mass $m$ is initially moving to the right with speed $v$ toward a series of hills and valleys on a frictionless surface, as shown. What range of speeds must the mass have to become trapped in the valley between points $A$ and $B$?

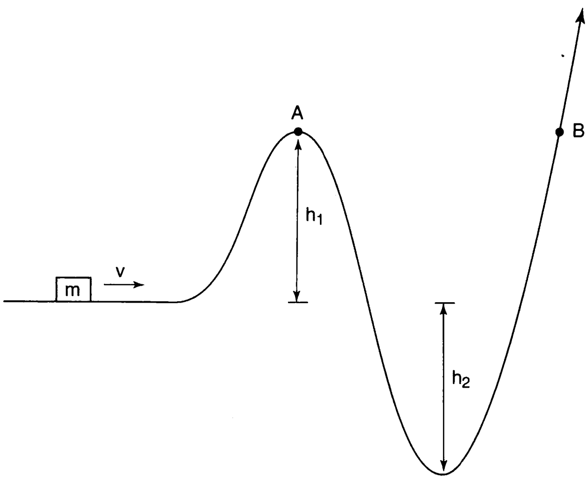
(A) $v > \sqrt{2gh_1}$

(B) $\sqrt{2g(h_1 + h_2)} > v > \sqrt{2gh_1}$

(C) $\sqrt{2gh_1h_2/(h_1 + h_2)} > v > \sqrt{2gh_1}$

(D) $\sqrt{2g(h_1^2 + h_2^2)^{1/2}} > v > \sqrt{2gh_1}$

(E) It is impossible for the mass to become trapped.




**Question 24**



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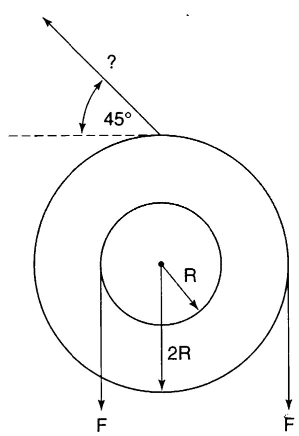
25. If a constant net force $F$ causes an object to accelerate from rest to a final velocity of $v$ while moving a distance $\Delta x$, what is the object's mass?

(A) $2\Delta xF/v_f^2$
(B) $2\Delta xF/v_f$
(C) $\Delta xF/v_f^2$
(D) $\Delta xF/v_f$
(E) Not enough information is given.

26. Two concentric cylinders of radii $R$ and $2R$ are fastened together and can rotate about their centers as in the figure. If two forces of magnitude $F$ are exerted on the cylinders as shown, what must the magnitude of the third unknown force be if the cylinders are to rotate with constant angular speed?

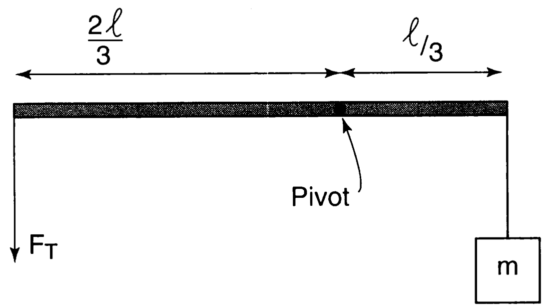
(A) $F/2$
(B) $F/\sqrt{2}$
(C) $F$
(D) $\sqrt{2}F$
(E) $2F$




**Question 26**

27. A rod of uniform mass density and total mass $M$ with a mass $m$ attached to one end is pivoted as shown. What is the tension force $F_T$ needed to prevent the rod from moving?

(A) $F_T = 2mg$
(B) $F_T = \frac{1}{2}mg - \frac{1}{4}Mg$
(C) $F_T = 2mg + \frac{1}{4}Mg$
(D) $F_T = \frac{1}{4}Mg - 2mg$
(E) $F_T = 2mg - \frac{1}{6}Mg$




**Question 27**

28. A child riding in a car along a flat road tosses a ball straight up with a speed $v$. To an observer standing on the ground outside the car, the ball appears to be thrown with an angle of elevation of $\theta$. What is the speed of the car?

(A) $v\sin\theta$
(B) $v\cos\theta$
(C) $v\sec\theta$
(D) $v\tan\theta$
(E) $v\cot\theta$

GO ON TO THE NEXT PAGE

29. A car of mass 400 kg is rounding a flat curve of radius 80 m. If the coefficient of static friction between the car's tires and the ground is 0.5, the maximum speed at which the car can round the curve is closest to which of the following?

(A) 10 m/s
(B) 20 m/s
(C) 30 m/s
(D) 40 m/s
(E) 80 m/s

30. If the acceleration of an object of mass $m$ starting from rest is given as a function of time by $a(t) = A\sqrt{t}$, then the kinetic energy of this object is proportional to

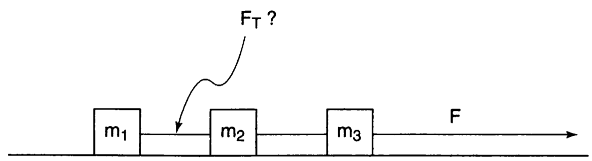
(A) $\dfrac{mA^2}{t}$
(B) $mA^2t$
(C) $mA^2t^3$
(D) $mA^2t^{-3}$
(E) $mA^2t^{3/2}$

31. A mass $m$ is thrown off the roof of a building of height $h$ with an initial speed $v_0$ and angle of elevation $\theta$ *below* the horizontal. What is the object's speed when it reaches the ground? (Do not ignore air friction.)

(A) $v \geq \sqrt{gh + v_0^2/2}$
(B) $v < \sqrt{gh + v_0^2/2}$
(C) $v < \sqrt{2gh + v_0^2}$
(D) $v < \sqrt{gh + v_0^2 \sin^2\theta/2}$
(E) $v < \sqrt{2gh + v_0^2 \sin^2\theta}$

32. A force **F** is exerted on the rightmost of three masses tied together by strings as shown, causing the masses to accelerate to the right over a frictionless horizontal surface. What is the magnitude of the indicated tension force?

(A) $\dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
(B) $\dfrac{m_1 F}{m_1 + m_2 + m_3}$
(C) $F/3$
(D) $\dfrac{F(m_1 - m_2 - m_3)}{m_1 + m_2 + m_3}$
(E) $\dfrac{F(m_2 + m_3 - m_1)}{m_1 + m_2 + m_3}$




**Question 32**

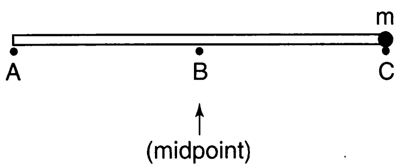
33. A simple pendulum of length $l$ has period $T$. A second pendulum of length $l/2$ is located at twice the distance from the center of Earth compared to the first pendulum. What is the period of the second pendulum?

(A) $T/2$
(B) $T/\sqrt{2}$
(C) $T$
(D) $T\sqrt{2}$
(E) $2T$

GO ON TO THE NEXT PAGE

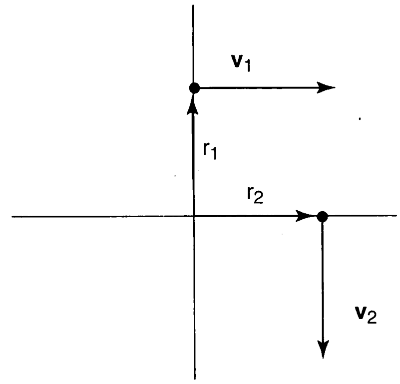
34. A mass $m$ is attached to a rod with uniform mass density at point $C$ as shown. The rotational inertia of this system is greatest when it is rotated about an axis perpendicular to the rod that passes through

(A) point $A$
(B) point $B$
(C) point $C$
(D) a point between points $A$ and $B$
(E) a point between points $B$ and $C$

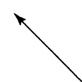


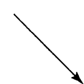

**Question 34**

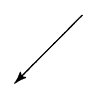
35. A particle's position and velocity vectors at two different times are shown. In what direction does the average acceleration vector point?

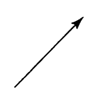



(A) 

(B) 

(C) 

(D) 

(E) 

**Question 35**

GO ON TO THE NEXT PAGE

## FREE-RESPONSE QUESTIONS

**Directions:** Use separate sheets of paper to write down your answers to the free-response questions. You may refer to the formula sheet in the Appendix (pages 639–642).

## Mechanics

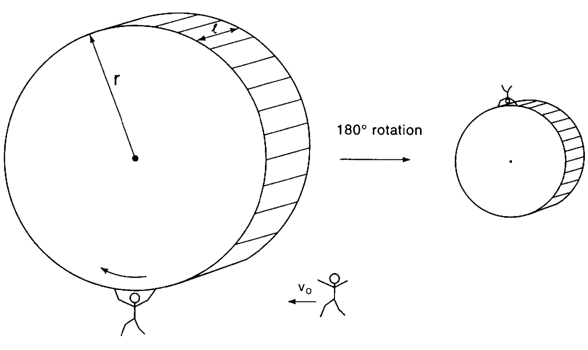
### MECHANICS I

A person (mass $m$) is flying to the left at a speed $v_0$ when she manages to grab onto the rim of a cylinder of radius $R$, length $l$, mass $m$, and uniform mass density that can rotate without friction about its axis (see figure).

(a) Using integration, calculate the rotational inertia of the cylinder.

(b) What is the person's linear speed the instant after she grabs the cylinder?

(c) If the collision between the person and the cylinder lasts a time $\Delta t$, what is the minimum force the person's hands must be capable of sustaining? (Assume $\Delta t$ is very small, so that the person's height does not change significantly during this time interval.)




**Question I**

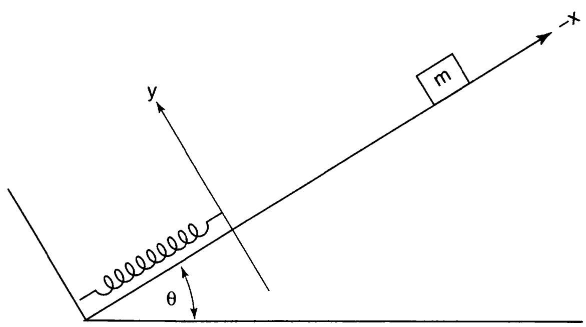
Then the cylinder rotates 180°, so that the person is at the top of the cylinder.

(d) What is the angular velocity at that instant?

(e) What force must the person's hands sustain now? (Assume that the wheel exerts a downward force on the person in order to keep her from flying off the wheel.)

### MECHANICS II

A mass $m$ slides down a frictionless incline and sticks to a spring (spring constant $k$), initiating simple harmonic motion (see figure).




**Question II**

(a) Express the potential energy $U$ of the oscillating mass and spring system in terms of $x$ (given the axis as shown, where the origin lies at the spring's initial unstretched length).

(b) Graph this $U(x)$ function, indicating the points where $U(x) = 0$.

(c) Based solely on this $U(x)$ curve, where do you expect the equilibrium position of the simple harmonic motion to lie?

(d) Based solely on Newton's second law, where does the equilibrium point lie?

(e) What is the angular frequency of the SHM?

GO ON TO THE NEXT PAGE

**MECHANICS III**

A person of mass $m_p$ is standing in an elevator of mass $m_e$ suspended from the top of a building by a spring with spring constant $k$.

(a) What is the equilibrium extension of the spring?

(b) Suppose the elevator is displaced from equilibrium by a small distance. What is the resulting frequency of oscillation?

(c) What range of accelerations can the elevator experience without losing contact with the person?

(d) What is the maximum amplitude with which the system can oscillate without the elevator's losing contact with the person?

(e) Assuming the amplitude of the oscillations is maximized, where is the acceleration of the person

  (i) maximum?
  (ii) minimum?
  (iii) zero?

GO ON TO THE NEXT PAGE