AP® Physics C: Electricity and Magnetism 2015 Free-Response Questions

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$\left|\vec{F}_f\right| \le \mu\left|\vec{F}_N\right|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{\max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_G\right| = \frac{Gm_1m_2}{r^2}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

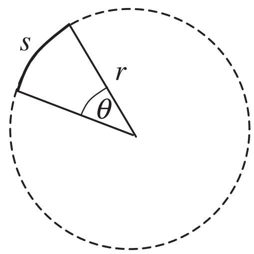
$a^2 + b^2 = c^2$

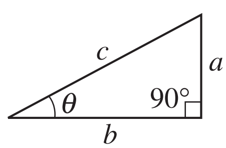
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

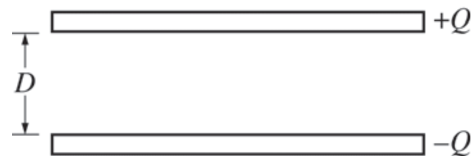
VECTOR PRODUCTS

$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

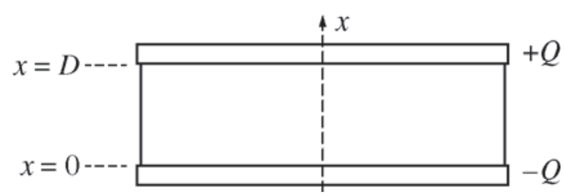
A parallel-plate capacitor is constructed of two parallel metal plates, each with area *A* and separated by a distance *D*. The plates of the capacitor are each given a charge of magnitude *Q*, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field *E* between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field *E* between the plates. Express your answer in terms of *A*, *D*, *Q*, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of *A, D*, *Q*, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of *x*.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

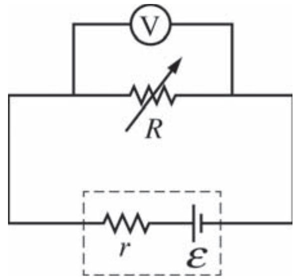
ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

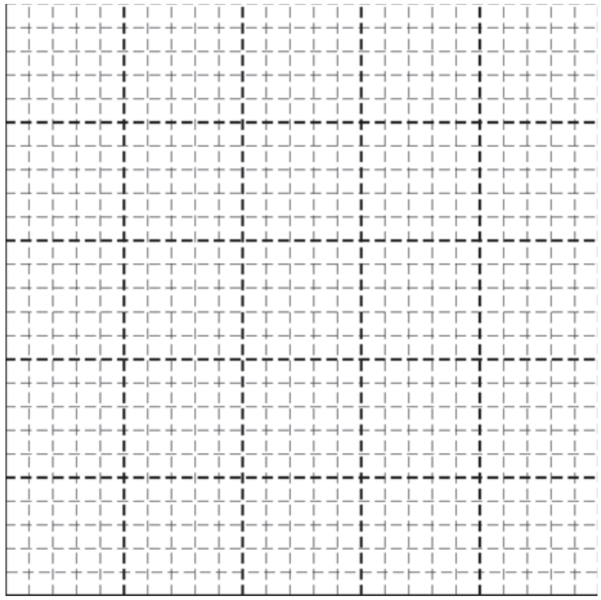
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ (1/Ω)	$1/V$ (1/V)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

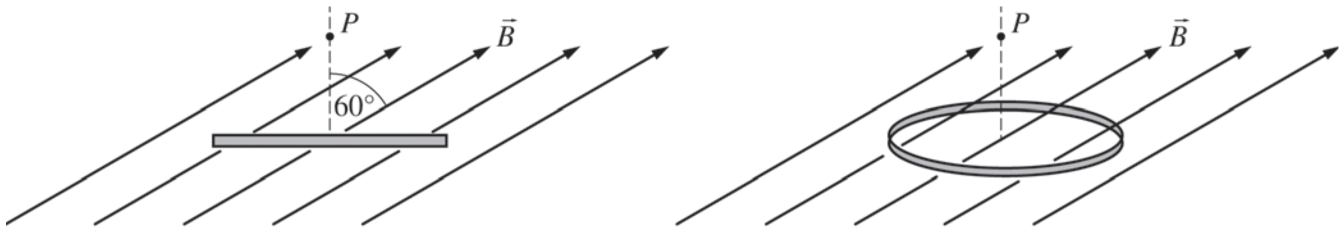
(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.



Edge View   Perspective View

E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where a = 4.0 T and b = 0.20 s^{-1}.

(a) Derive an expression for the magnetic flux through the loop as a function of time t.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point P ?

____ Clockwise   ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM

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$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

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$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

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$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

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$R = \frac{\rho \ell}{A}$

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$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

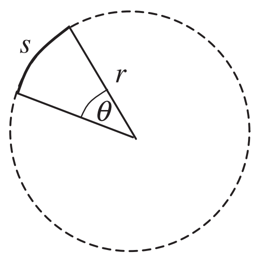
$a^2 + b^2 = c^2$

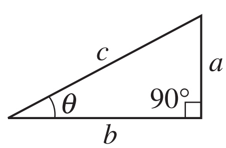
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

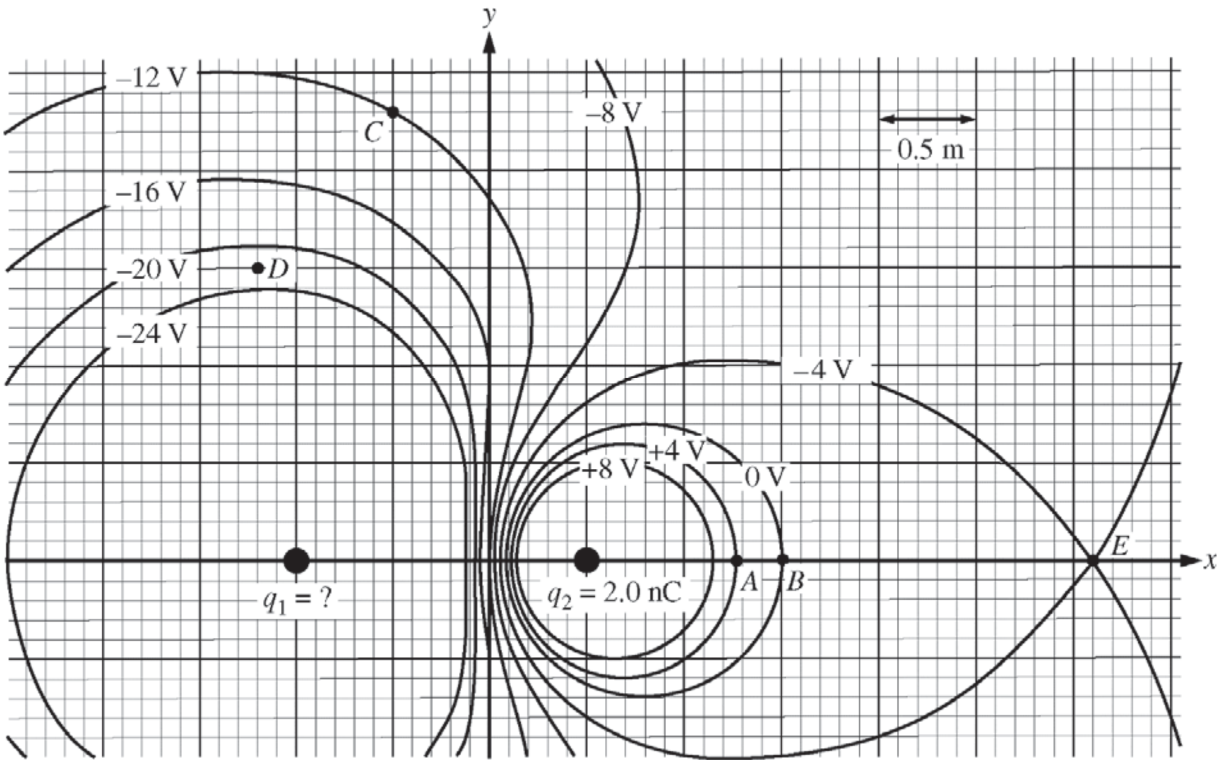
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of q_1.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

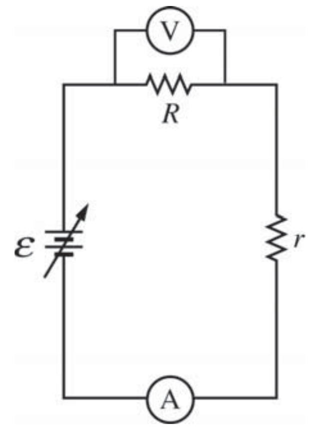
____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.



E&M.2.

The circuit shown above consists of a source of variable emf $\mathcal{E}$, an ideal ammeter A, an ideal voltmeter V, a resistor of resistance *R*, and a sample of wire with resistance *r*.

(a) How does the current through the wire sample compare with the current through the resistor *R* ?

_____ It is greater through *R*. _____ It is greater through the sample.

_____ It is the same through both. _____ It depends on the resistance of the sample.

Justify your answer.

(b) How does the potential difference across the wire sample compare with the potential difference across the resistor *R* ?

_____ It is greater across *R*. _____ It is greater across the sample.

_____ It is the same across both. _____ It depends on the resistance of the sample.

Justify your answer.

With the sample of wire in place, the emf of the source is set to a given value. The current through and potential difference across the resistor *R* are measured. This is repeated for several values of emf, and the data are recorded in the table below.

$\mathcal{E}$ (V)	V_R (V)	I_R (A)		
0.250	0.179	0.162		
0.500	0.335	0.327		
0.750	0.520	0.490		
1.000	0.670	0.687		

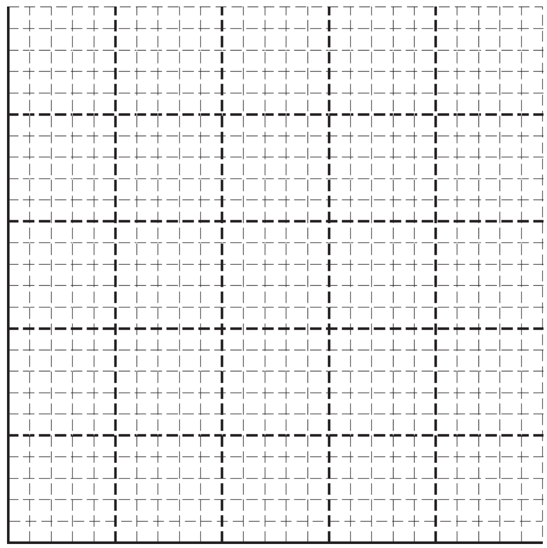
(c) Indicate below which quantities should be graphed to yield a straight line that could be used to calculate a numerical value for the resistance of the wire sample.

Horizontal axis: ____________

Vertical axis: ____________

You may use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given.

(d) On the grid below, plot the straight line data points from part (c). Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(e) Use your straight line to calculate the value of the resistance of the wire sample.

(f) The wire sample has a length of 3.00 m and a radius of 1.00×10^{-3} m . Calculate the resistivity of the material from which the wire sample is made.

(g)

i. Suppose the ammeter used to collect these data was not ideal. Would the actual value of the resistance of the wire sample be greater than, less than, or equal to that calculated in part (e) ?

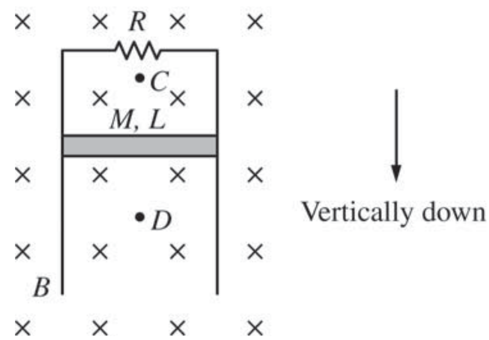
____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. If the ideal voltmeter is replaced by a voltmeter that is not ideal and the experiment is repeated, would the readings of the ideal ammeter be greater than, less than, or equal to those in the data chart before part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left   ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than   _____ Less than   _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than   _____ Less than   _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

STOP

END OF EXAM

2017

AP® CollegeBoard

AP Physics C: Electricity and Magnetism

Free-Response Questions

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23} \text{ mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11} \left(\text{N}\cdot\text{m}^2\right)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8 \text{ m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\left(\text{N}\cdot\text{m}^2\right)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9} \left(\text{N}\cdot\text{m}^2\right)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ (T}\cdot\text{m)/A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7} \text{ (T}\cdot\text{m)/A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5} \text{ N/m}^2 = 1.0 \times 10^{5} \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu|\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1m_2}{r^2}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
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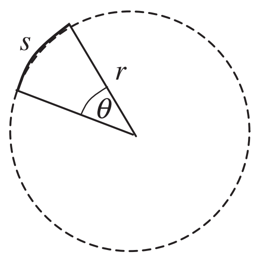

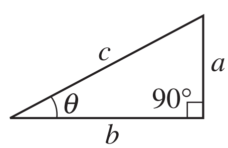

CALCULUS

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$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

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$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln\left|x+a\right|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

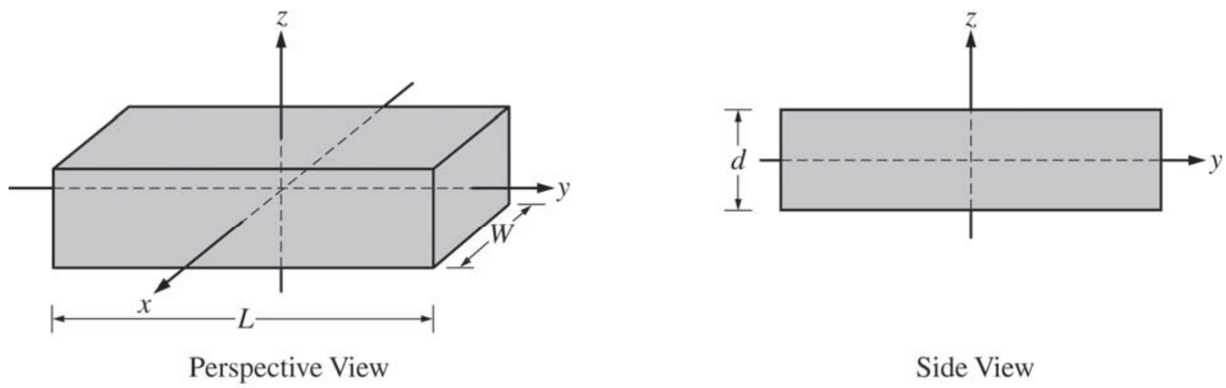
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

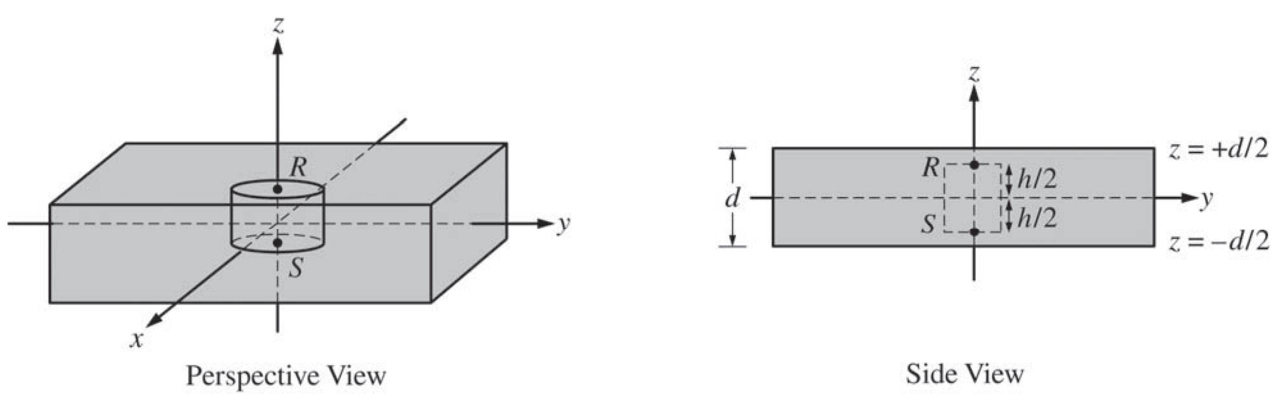
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the *xy*-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the *z*-axis, as shown in the figure below.



Note: Figures not drawn to scale.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

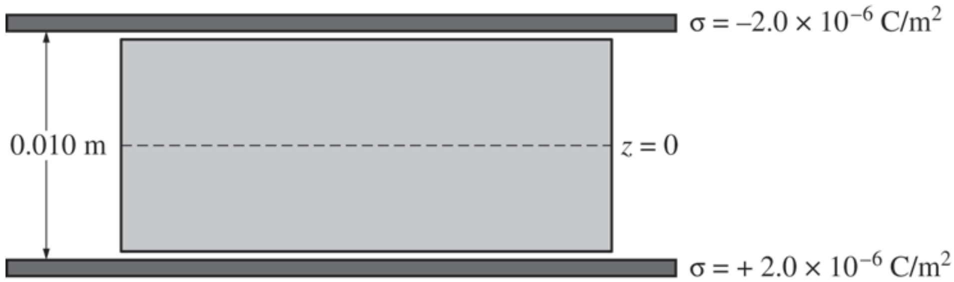
i. • R

ii. • S

b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.



Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \text{C/m}^2$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \text{C/m}^3$.

(c)

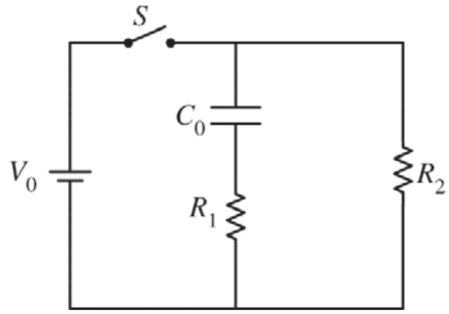
i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

____ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

Justify your answer.

ii. Calculate the value z_0.

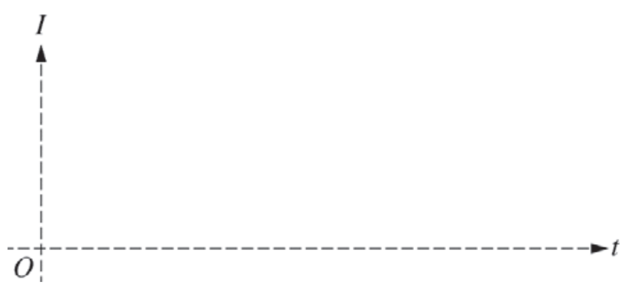
(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.



2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

   (a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

   (b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

   (c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

   ____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

   Justify your answer.

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

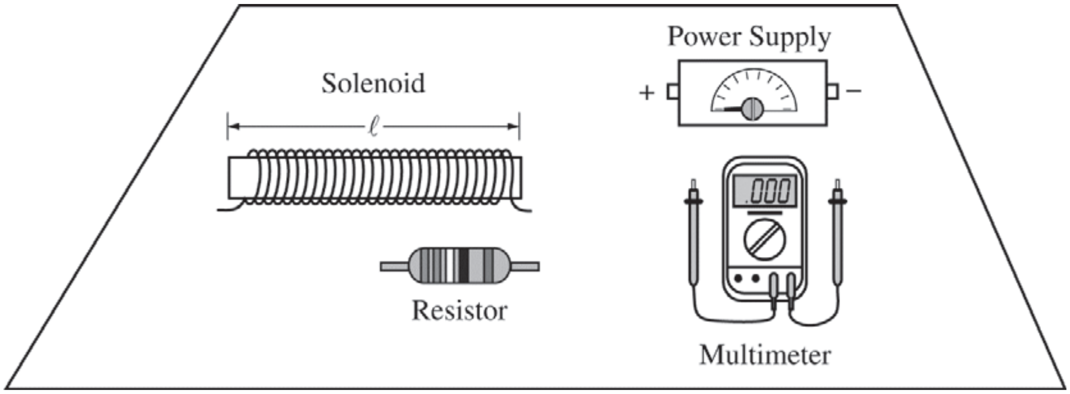
____ Increasing ____ Decreasing ____ Constant

Justify your answer.

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.



3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(a)

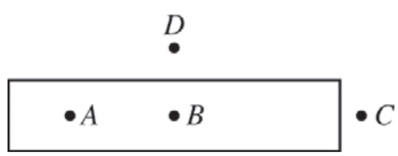
i. On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.

ii. Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

____ Toward the top of the page   _____ To the left   ____ Out of the page

____ Toward the bottom of the page   _____ To the right   ____ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A, B, and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B.

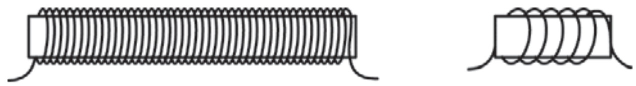


iii. From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0.

____ A ____ B ____ C ____ D

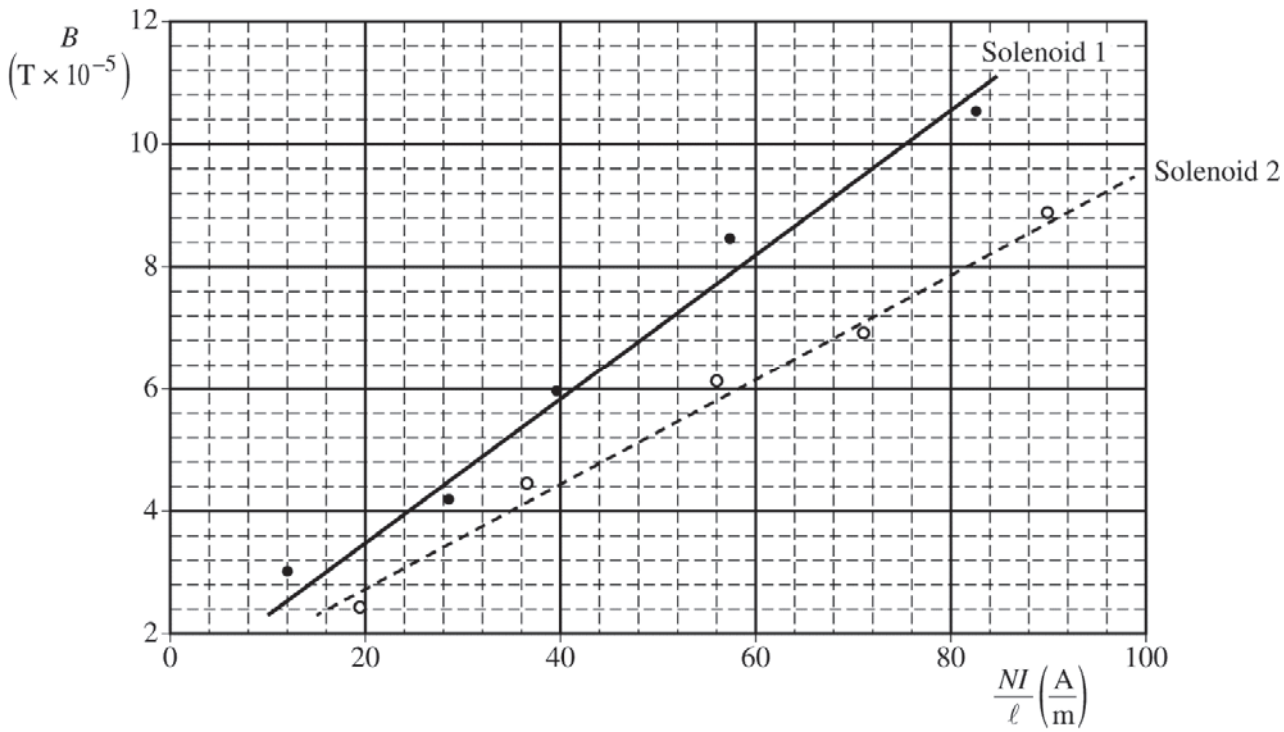
Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



Note: Figures not drawn to scale.

A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.



(b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0?

____ Solenoid 1 ____ Solenoid 2

Justify your answer.

(c)

i. Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0.

ii. Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

GO ON TO THE NEXT PAGE.

(d)

i. What is a reasonable physical explanation for a best-fit line that does not pass through the origin?

ii. Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

STOP

END OF EXAM

2018

AP CollegeBoard

AP Physics C: Electricity and Magnetism

Free-Response Questions

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^3 \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5\ \text{N/m}^2 = 1.0 \times 10^5 \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu|\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1m_2}{r^2}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

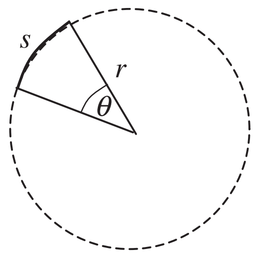
$a^2 + b^2 = c^2$

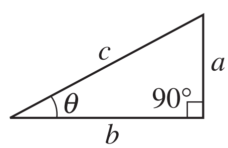
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

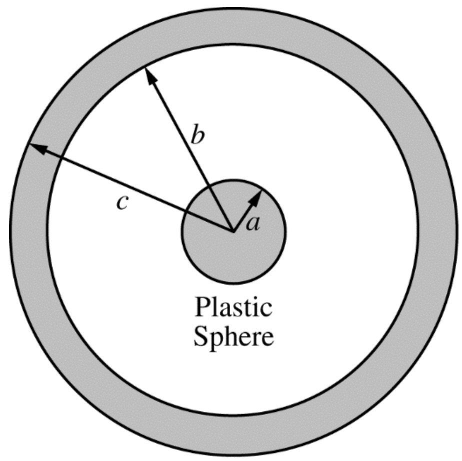
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β , r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

      i. $r < a$

      ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

      i. Determine the charge on the inner surface of the conducting shell.

      Justify your answer.

      ii. Determine the charge on the outer surface of the conducting shell.

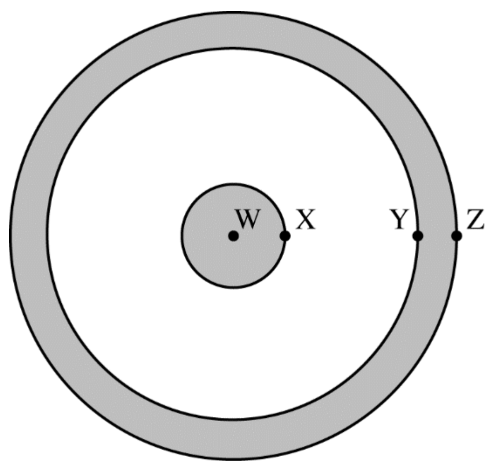
GO ON TO THE NEXT PAGE.

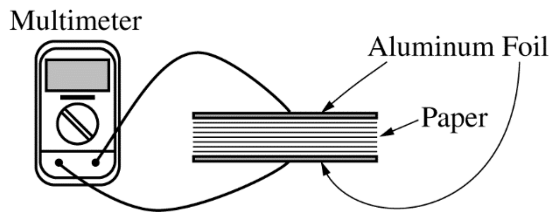
(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z





2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

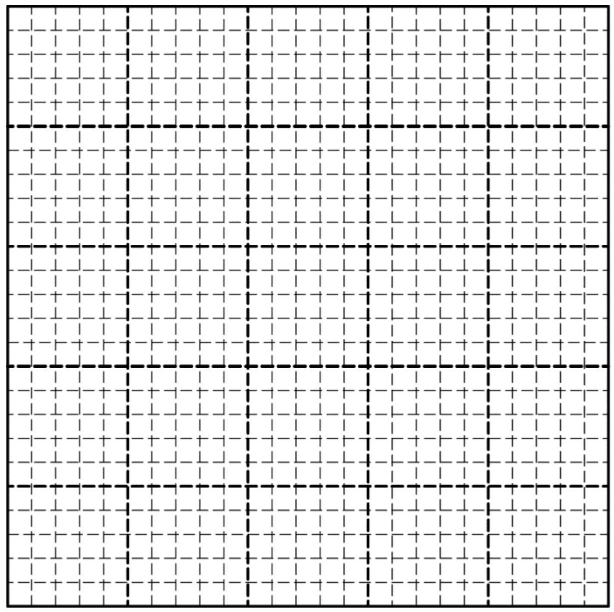
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

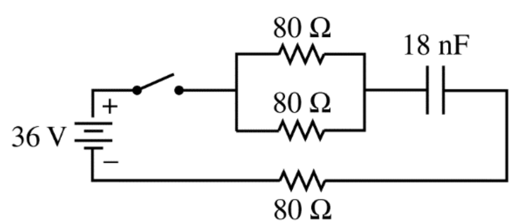
Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

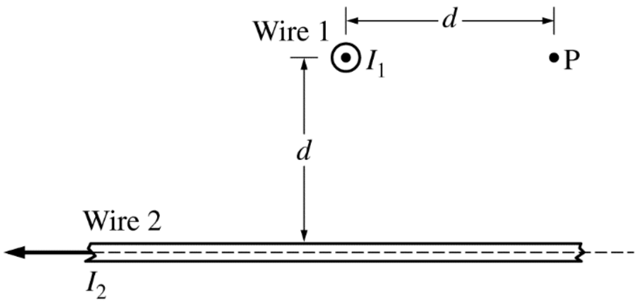


Figure 1. Side view

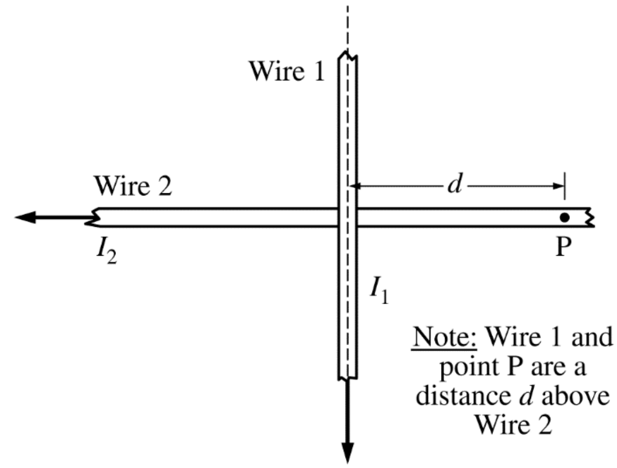


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

   (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

   (b) Derive an expression for the magnitude of the net magnetic field at point P.

   (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

   (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

   ____ Wire 1 will not move.

   ____ Wire 1 will move upward as viewed in Figure 1.

   ____ Wire 1 will move downward as viewed in Figure 1.

   ____ Wire 1 will rotate clockwise as viewed in Figure 2.

   ____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

   Justify your answer.

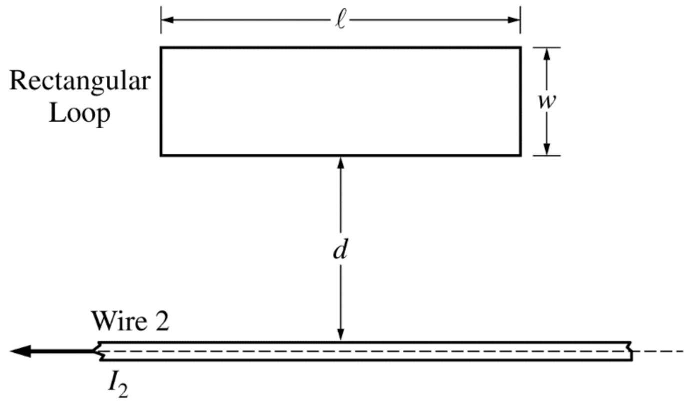


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise   ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM

2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Free-Response Questions Set 1

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}
Universal gas constant,	$R = 8.31$ J/(mol•K)
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K
Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19}$ J
Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ $(\text{N•m}^2)/\text{kg}^2$
Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
	$hc = 1.99 \times 10^{-25}$ J•m $= 1.24 \times 10^{3}$ eV•nm
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}$ $\text{C}^2/(\text{N•m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}$ $(\text{N•m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5}$ N/m^2 $= 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	$0°$	$30°$	$37°$	$45°$	$53°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu|\vec{F}_N|$

$\Delta E = W = \int \vec{F} \bullet d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \bullet \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum\vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1m_2}{r^2}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \bullet d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \bullet d\vec{A}$

$\mathcal{E} = \oint \vec{E} \bullet d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

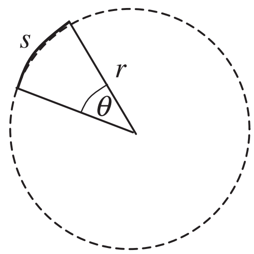
$a^2 + b^2 = c^2$

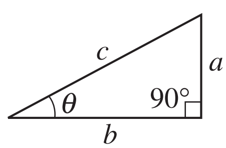
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

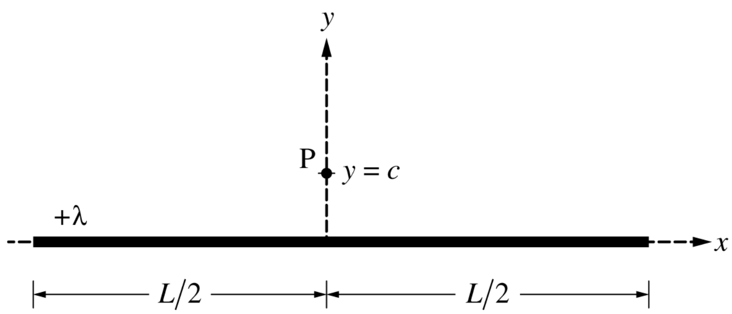
$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

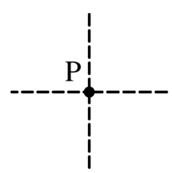


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

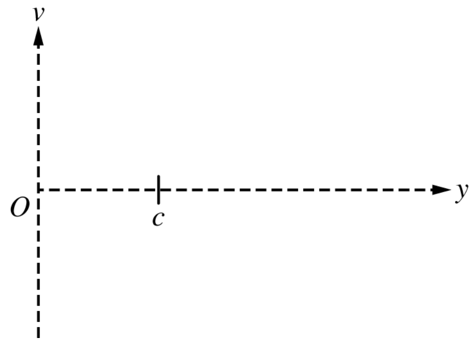
i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

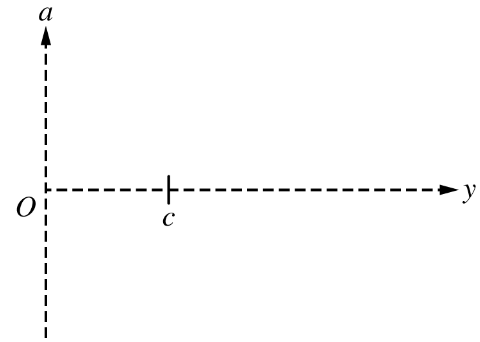


ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

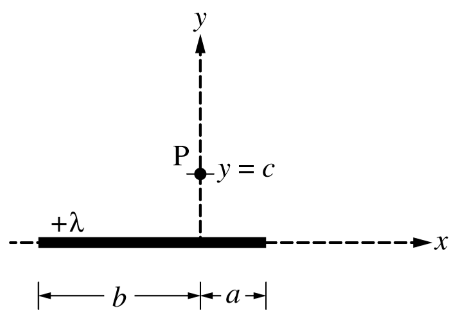
iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of <u>position</u> y and the acceleration a as a function of <u>position</u> y for the proton.

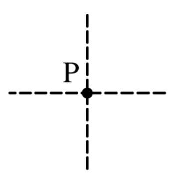




The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

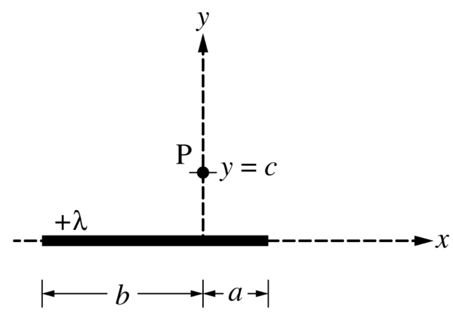


(d)

i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $$\left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{\left(c^2 + x^2\right)^{3/2}}\, dx$$

Vertical component: $$\left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{\left(c^2 + x^2\right)}\, dy$$

(e)

i. One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

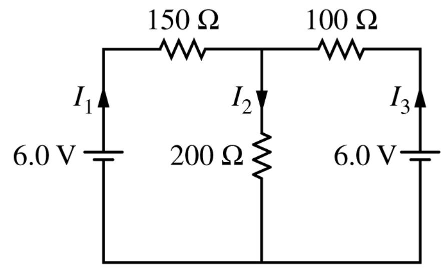


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

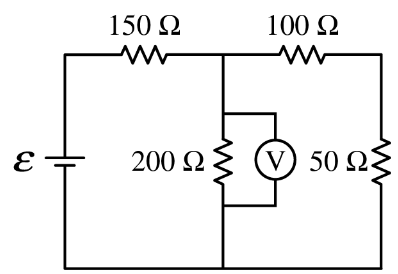


Figure 2

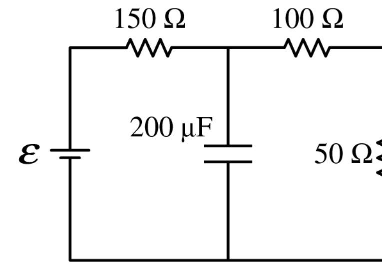
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

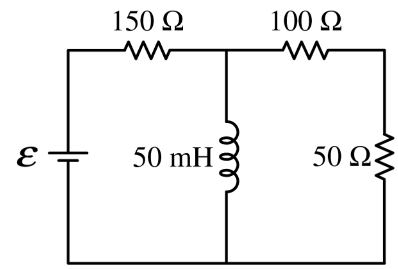
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

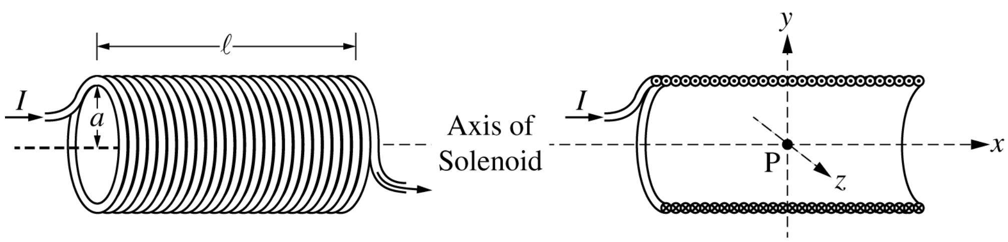


ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?



____ Greater than ____ Less than ____ Equal to

Justify your answer.



Solenoid   Cutaway View

Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) Select the correct direction of the magnetic field at point P.

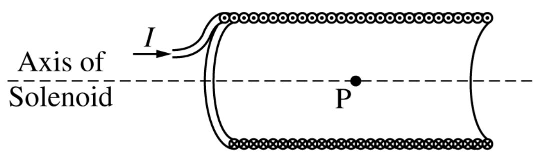
____ $+x$-direction ____ $+y$-direction ____ $+z$-direction

____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

(b)

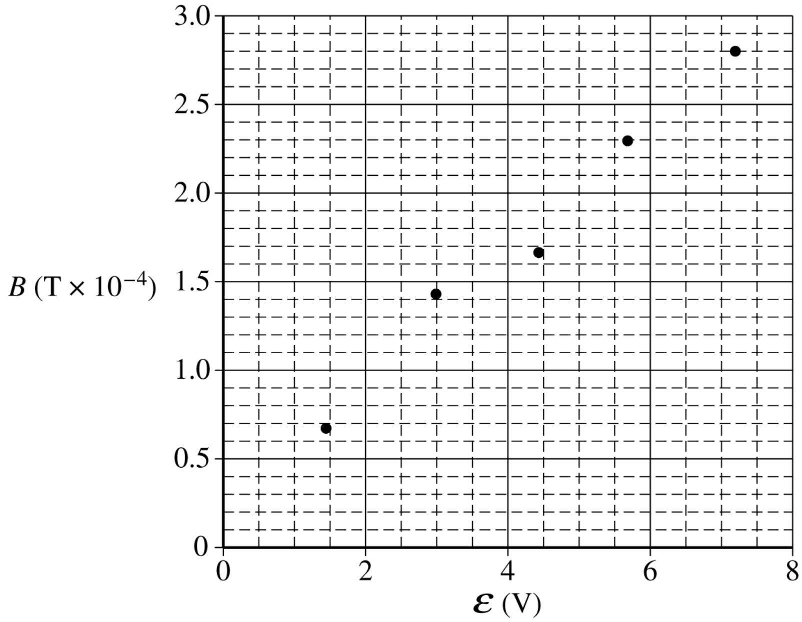
i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

i. On the graph above, draw a best-fit line for the data.

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

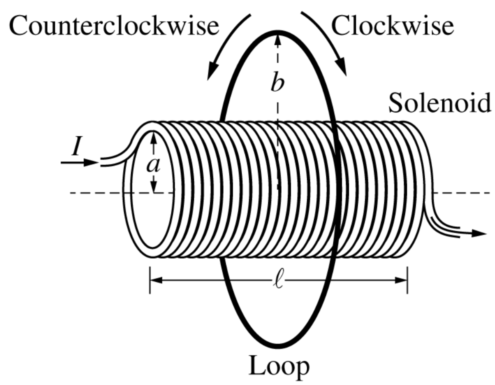
____ Yes ____ No

Justify your answer.

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

STOP

END OF EXAM

2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Free-Response Questions Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5\ \text{N}/\text{m}^2 = 1.0 \times 10^5 \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	$0°$	$30°$	$37°$	$45°$	$53°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

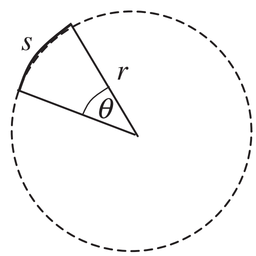
$a^2 + b^2 = c^2$

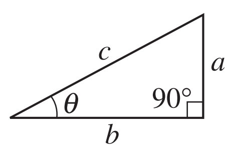
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x + a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

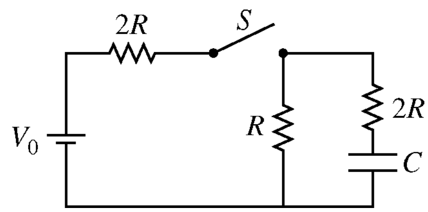
VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.

   (b) Derive an expression for the charge on the capacitor.

   (c) Derive an expression for the energy stored in the capacitor.

   Now the switch is opened at time $t = 0$.

   (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

(e)

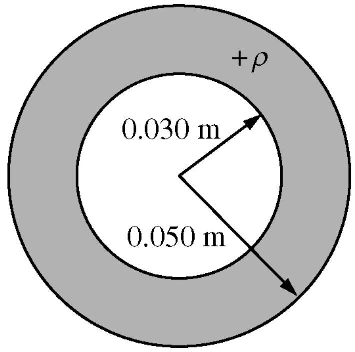
i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030\text{ m}: \quad \rho = 0$$

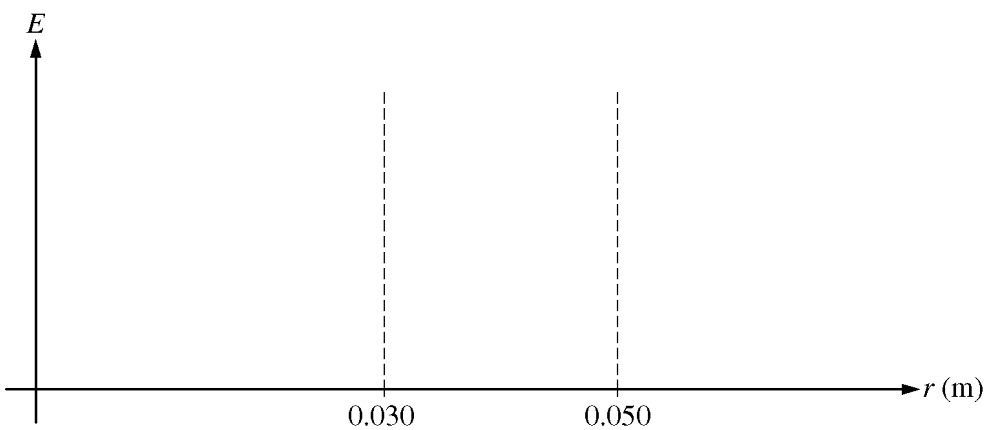
$$0.030\text{ m} < r < 0.050\text{ m}: \quad \rho = b/r\text{, where } b = 1.6 \times 10^{-6}\ \text{C/m}^2$$

$$r > 0.050\text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.

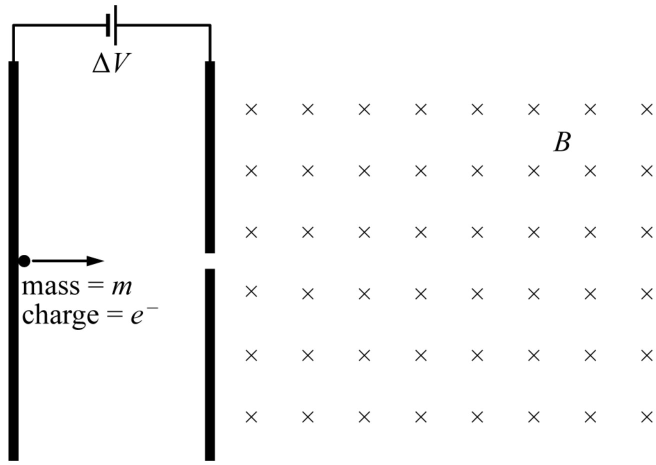


(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.



3. Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(a)

i. What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page

____ Toward the top of the page ____ Toward the bottom of the page

ii. Describe the path taken by the sphere after it enters the magnetic field.

(b) Derive an expression for the speed of the sphere as it passes through the small opening.

(c) Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field $\left(\mathrm{T} \times 10^{-3}\right)$	2.62	2.78	3.39	3.54	3.78	3.99

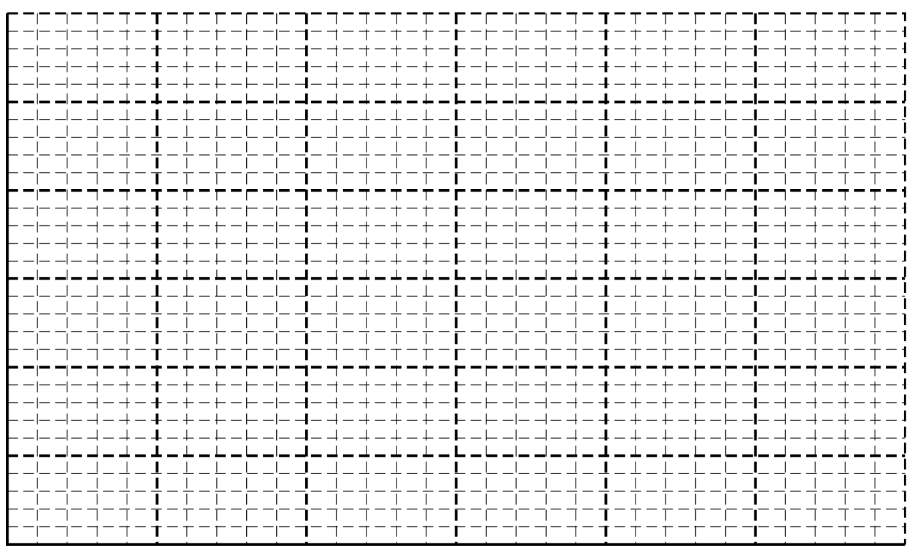
(d) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(e) On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



(f) Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

STOP

END OF EXAM

2021

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Free-Response Questions

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}
Universal gas constant,	$R = 8.31$ J/(mol•K)
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K
Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
1 electron volt,	1 eV $= 1.60 \times 10^{-19}$ J
Speed of light,	$c = 3.00 \times 10^8$ m/s
Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ $(\text{N•m}^2)/\text{kg}^2$
Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
1 unified atomic mass unit,	1 u $= 1.66 \times 10^{-27}$ kg $= 931$ MeV/c^2
Planck's constant,	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
	$hc = 1.99 \times 10^{-25}$ J•m $= 1.24 \times 10^3$ eV•nm
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}$ $\text{C}^2/(\text{N•m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ $(\text{N•m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	1 atm $= 1.0 \times 10^5$ N/m^2 $= 1.0 \times 10^5$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
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P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

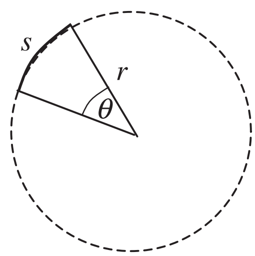
$a^2 + b^2 = c^2$

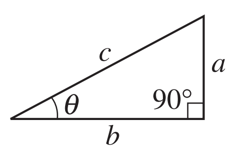
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

Begin your response to **QUESTION 1** on this page.

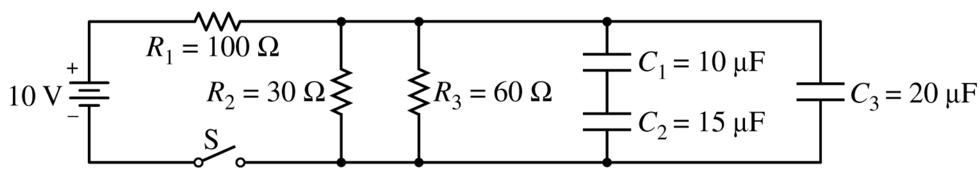
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(a) Calculate the current through R_1 immediately after switch S is closed.

Switch S has been closed for a long time, and the circuit has reached a steady state.

(b) Calculate the potential difference across R_1.

(c)
i. Calculate the charge stored on the positive plate of capacitor C_2.

ii. Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2 ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

Continue your response to **QUESTION 1** on this page.

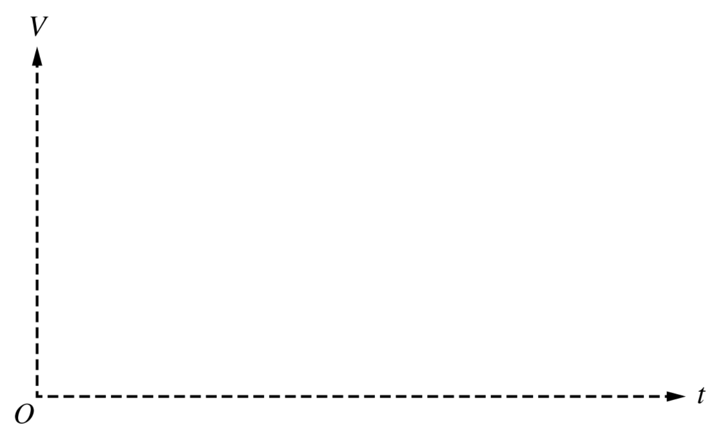
Switch S is then opened.

(d)

i. Determine the current through R_1 immediately after the switch is opened.

ii. Calculate the current through R_2 immediately after the switch is opened.

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time $t = 0$. Label the maximum value.



Capacitor C_3 is replaced by two $10\,\mu F$ capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time $t = 0$.

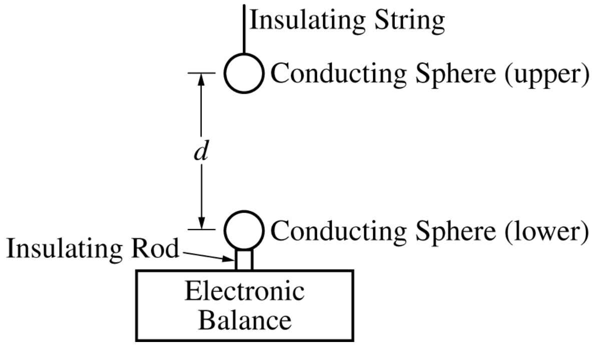
(f) For $t > 0$, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e) ?

_____ Above   _____ Below   _____ The same

Justify your answer.

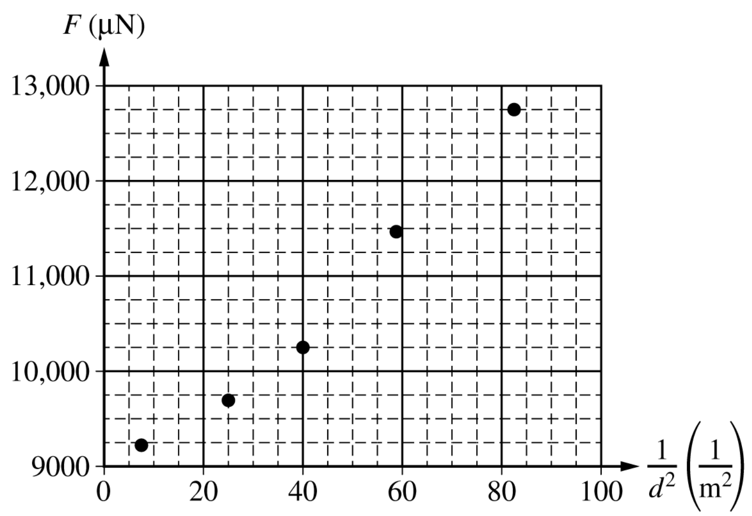
GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.



2. Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.



(a)

i. Draw a line that represents the best fit to the points shown.

ii. Use the graph to calculate the charge Q.

GO ON TO THE NEXT PAGE.

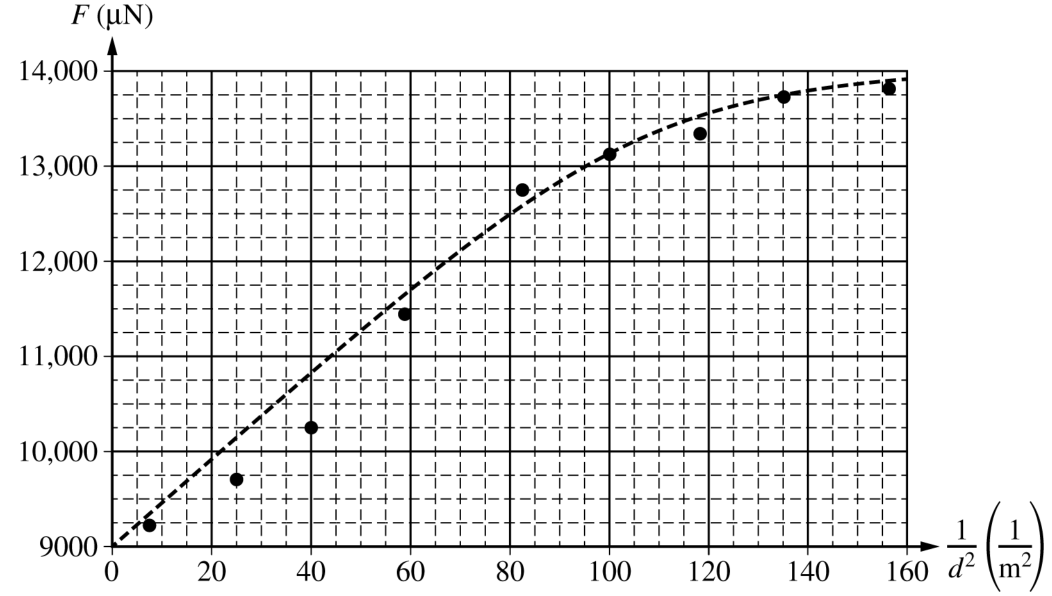
Continue your response to **QUESTION 2** on this page.

iii. On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

iv. Determine the distance between the centers of the spheres for the data point indicated above.

v. What physical quantity does the vertical intercept represent?

Justify your answer.



The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

(b) One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

____ Yes ______ No

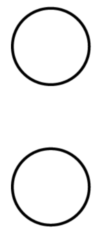
Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

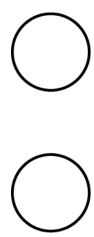
i. On the circles representing the spheres below, use a single “+” sign on each sphere to represent the locations of highest concentration of the excess positive charges.



ii. Explain how this rearrangement could be responsible for the discrepancy.

(d) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

i. On the circles representing the spheres below, use a single “+” sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single “–” sign on the top sphere to represent the location of the highest concentration of the excess negative charges.



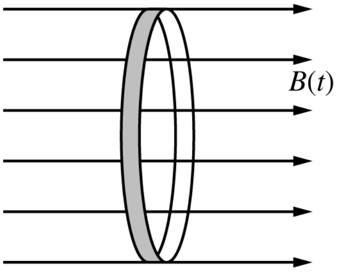
ii. For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

_____ Greater than   _____ Less than   _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1\text{ s}$, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \dfrac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1\text{ s}$. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

Assume $A = 0.50\text{ m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\text{ T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1\text{ s}$ to $t = 2\text{ s}$.

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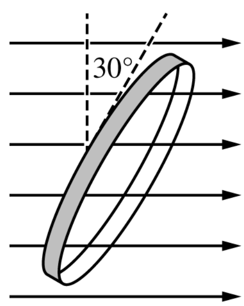
Continue your response to **QUESTION 3** on this page.



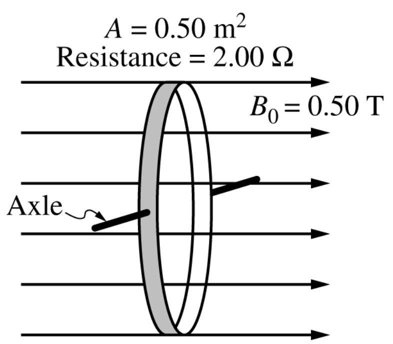
The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

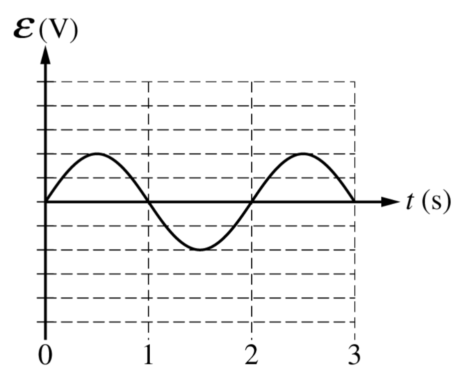
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1$ s according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(c) Will the amount of energy dissipated in the ring from $t = 1$ s to $t = 2$ s be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.





The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50$ T, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

(d) Calculate the angular speed ω of the rotating ring in rad/s.

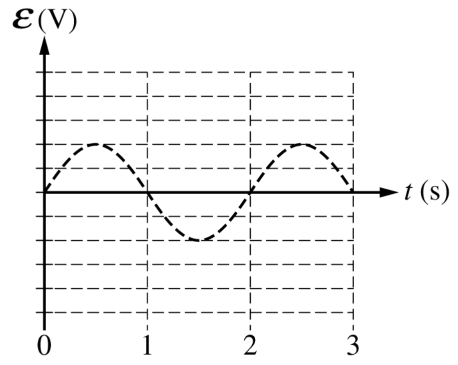
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

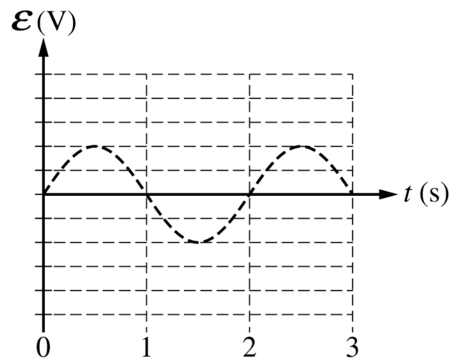
The ring now begins to rotate at an angular speed 2ω.

(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.



Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

STOP

END OF EXAM

2022

AP® Physics C: Electricity and Magnetism

Free-Response Questions
Set 1

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}\ \text{Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \bullet d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \bullet \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1m_2}{r^2}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \bullet d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \bullet d\vec{A}$

$\mathcal{E} = \oint \vec{E} \bullet d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

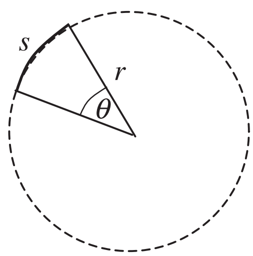
Circle
$A = \pi r^2$
$C = 2\pi r$
$s = r\theta$

Rectangular Solid
$V = \ell wh$

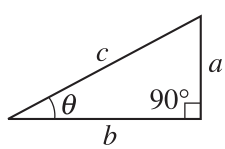
Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle



Right Triangle
$a^2 + b^2 = c^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

Begin your response to **QUESTION 1** on this page.

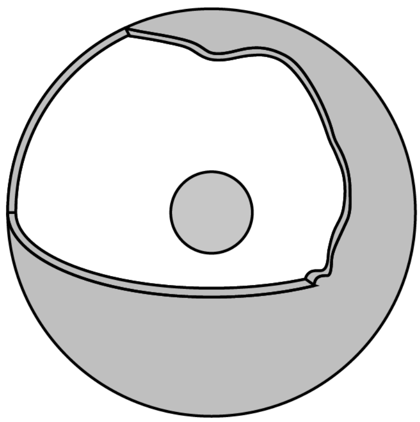
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

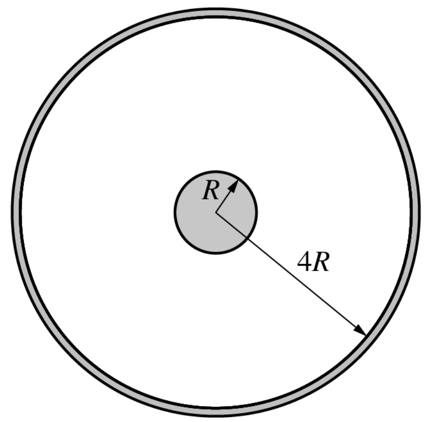
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View



Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

(c) The magnitude of the electric field at $r = R$ is $8\,\text{N/C}$. Calculate the value of the electric field at $r = 2R$.

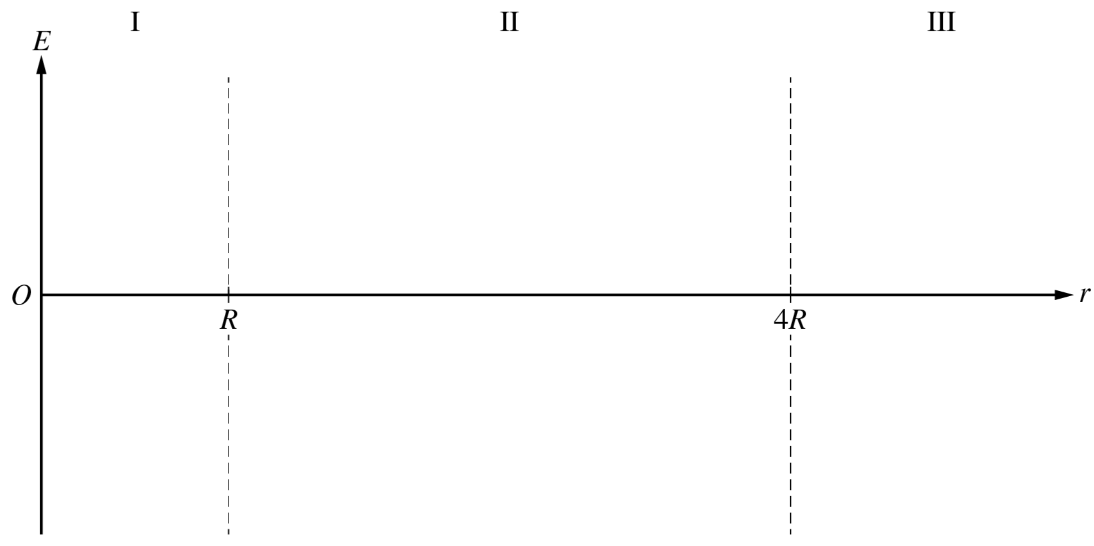
(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

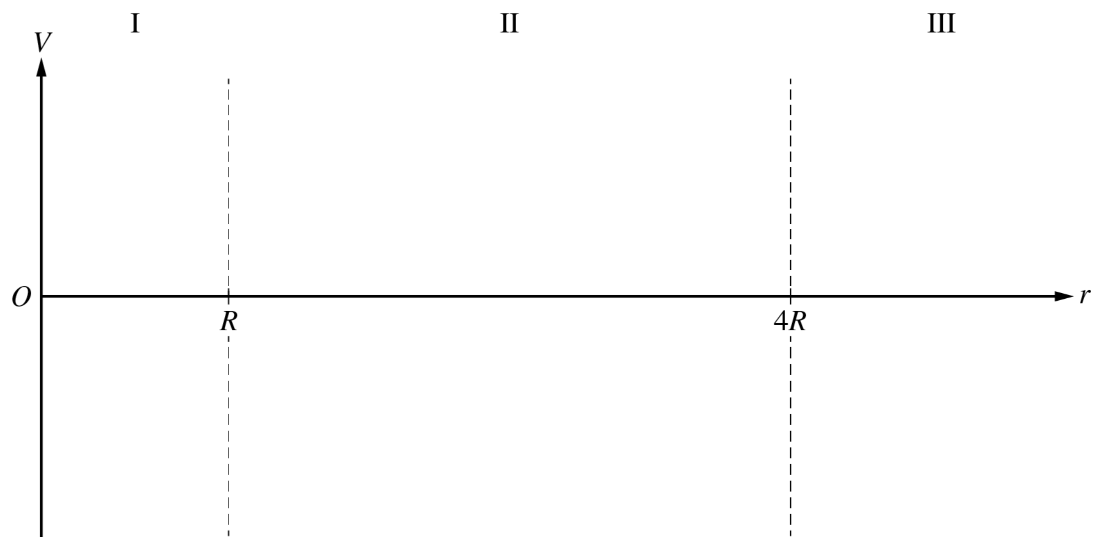
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



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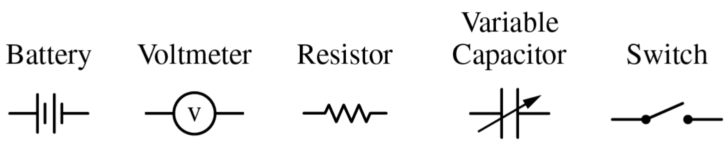
Begin your response to **QUESTION 2** on this page.

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

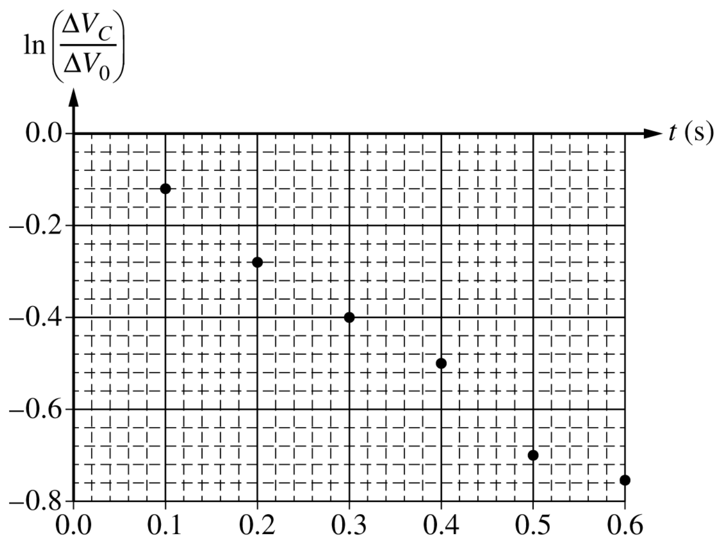
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

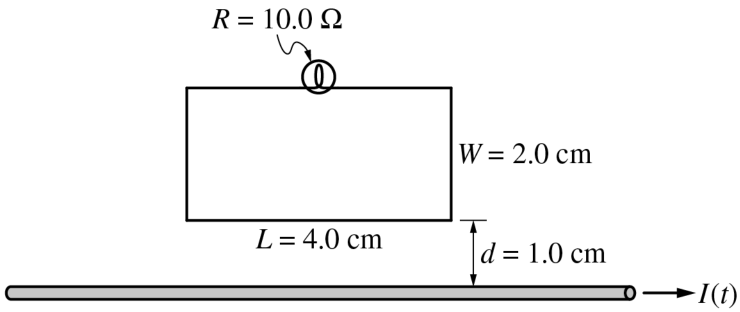
ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



3. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0$ cm and a width $W = 2.0$ cm and is positioned so one of the longer sides of the loop is a distance $d = 1.0$ cm above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. At time $t = 5.0$ s, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page   ___ Out of the page   ___ No direction, because the field is zero

Justify your answer.

(b) Calculate the magnetic flux through the loop due to only the long wire at time $t = 3.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the current through the lightbulb at time $t = 3.0$ s.

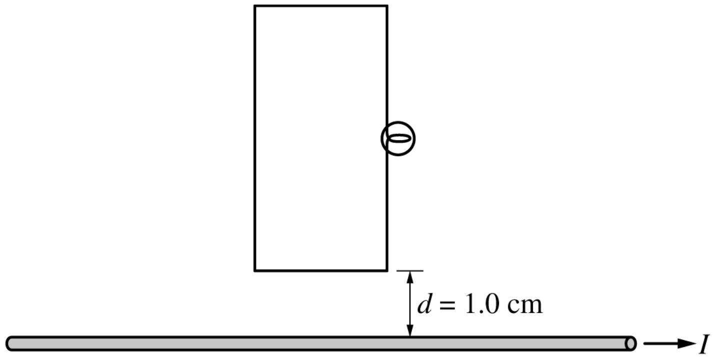
(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
___ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. The current through the lightbulb in the loop's new orientation at time $t = 3.0$ s is I_2. Which of the following correctly relates the current I_2 to I_1, the current through the lightbulb in part (c)?

___ $I_2 < I_1$ ___ $I_2 = I_1$ ___ $I_2 > I_1$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

2022

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AP® Physics C: Electricity and Magnetism

Free-Response Questions
Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu|\vec{F}_N|$

$\Delta E = W = \int \vec{F}\bullet d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F}\bullet\vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r}\times\vec{F}$

$\vec{\alpha} = \frac{\sum\vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r}\times\vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1m_2}{r^2}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint\vec{E}\bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int\vec{E}\bullet d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i\frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i\frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i\frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v}\times\vec{B}$

$\oint\vec{B}\bullet d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell}\times\hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell}\times\vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int\vec{B}\bullet d\vec{A}$

$\mathcal{E} = \oint\vec{E}\bullet d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

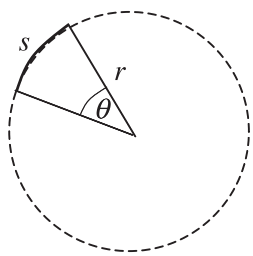
$a^2 + b^2 = c^2$

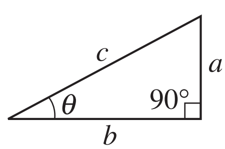
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$

Begin your response to **QUESTION 1** on this page.

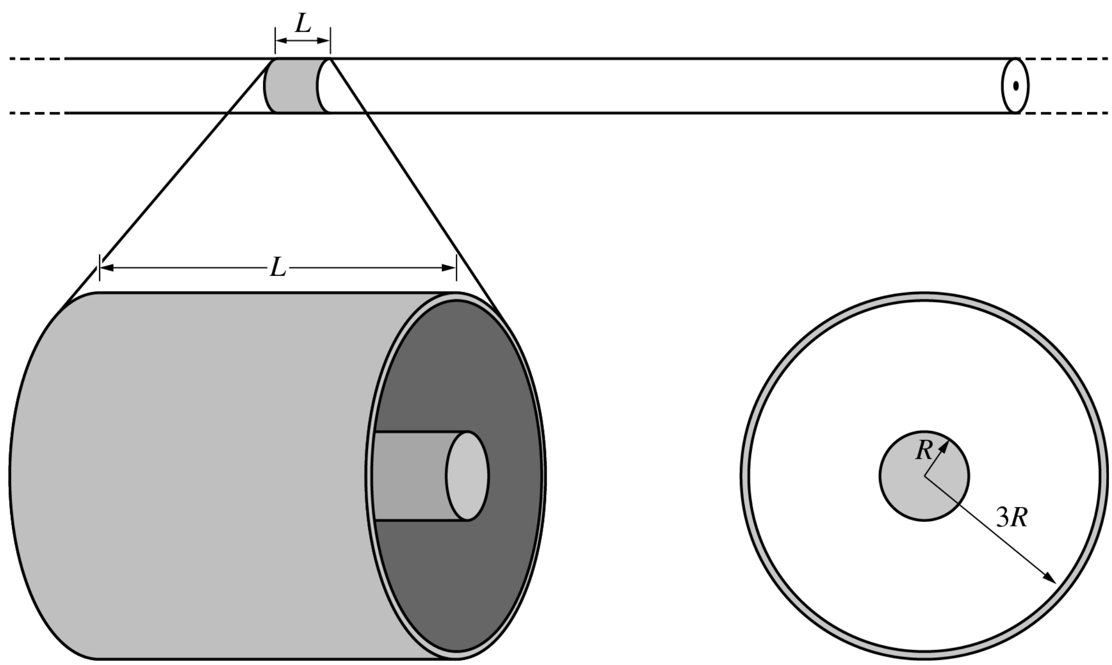
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is $12\ \text{N/C}$. Calculate the value of the electric field at $r = 2R$.

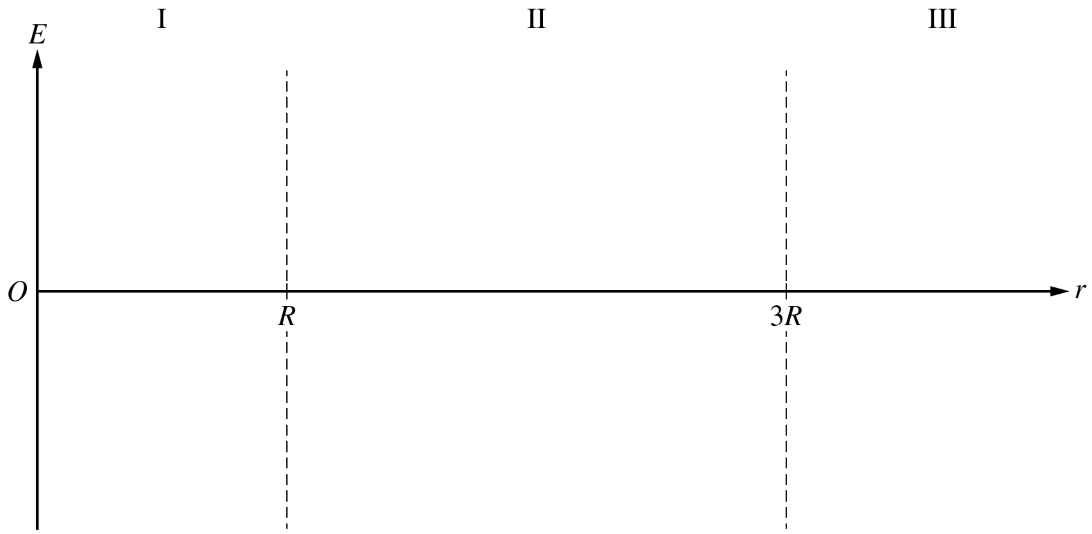
(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

GO ON TO THE NEXT PAGE.

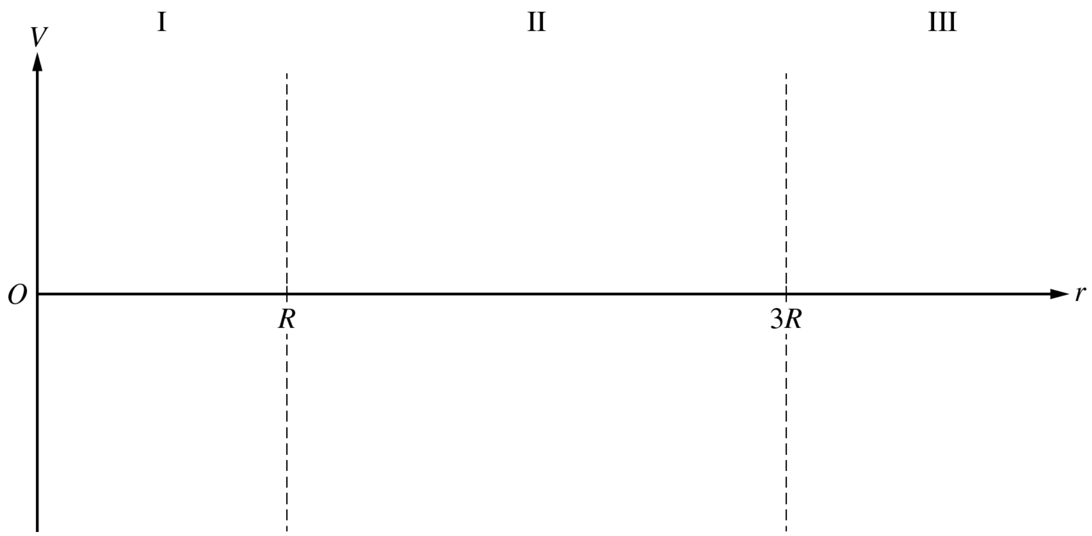
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



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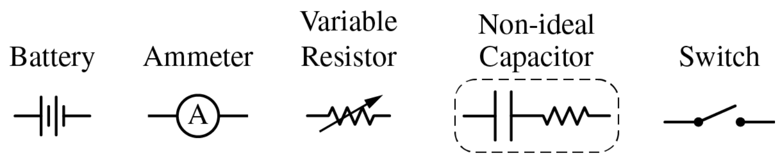
Begin your response to **QUESTION 2** on this page.

2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

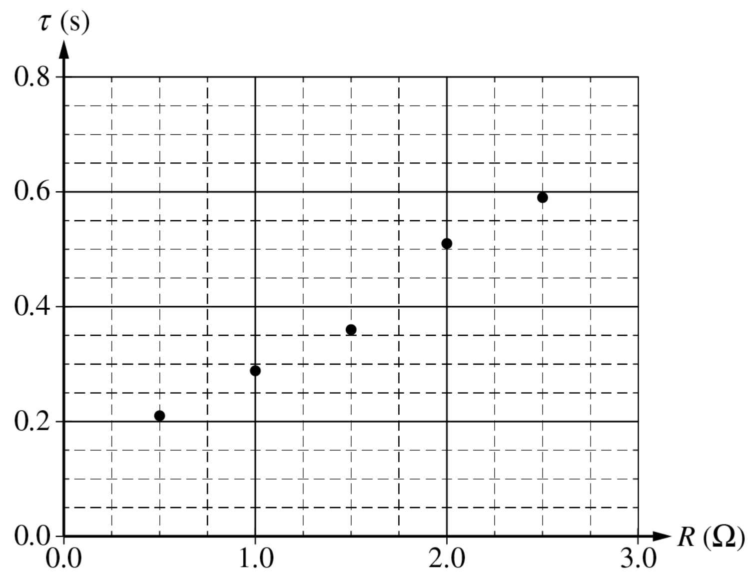
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the internal resistance r_C of the capacitor.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance r_C for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

___ Greater than ___ Less than ___ Equal to

Briefly justify your answer using features of the graph in part (c).

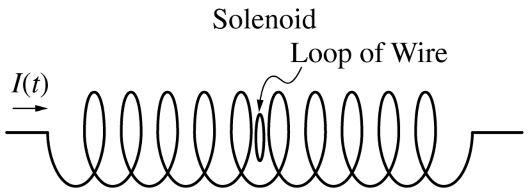
(e) The values of the variable resistor in the original experiment ranged from $0.5\ \Omega$ to $2.5\ \Omega$. The experiment is repeated with values ranging from $3.0\ \Omega$ to $6.0\ \Omega$. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

___ More steep ___ Less steep ___ Remain unchanged

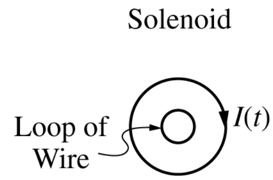
Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



Side View



End View

Note: Figures not drawn to scale.

3. A single loop of wire with resistance $3.0\ \Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0$ s, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

(b) Calculate the current in the loop of wire at time $t = 2.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\text{ s}$.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\text{ s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than 0.10 m.

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

2023

AP

AP® Physics C: Electricity and Magnetism

Free-Response Questions

Set 1

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

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Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
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UNIT SYMBOLS								
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	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
ε = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\varepsilon = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\varepsilon = -L \frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

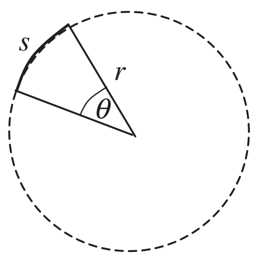
$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle



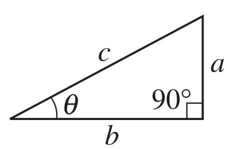
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

Begin your response to **QUESTION 1** on this page.

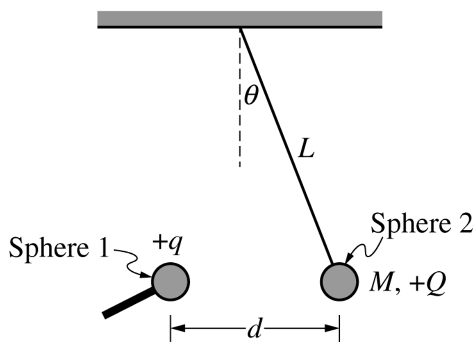
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

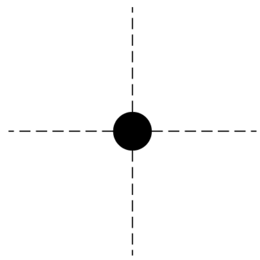


1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



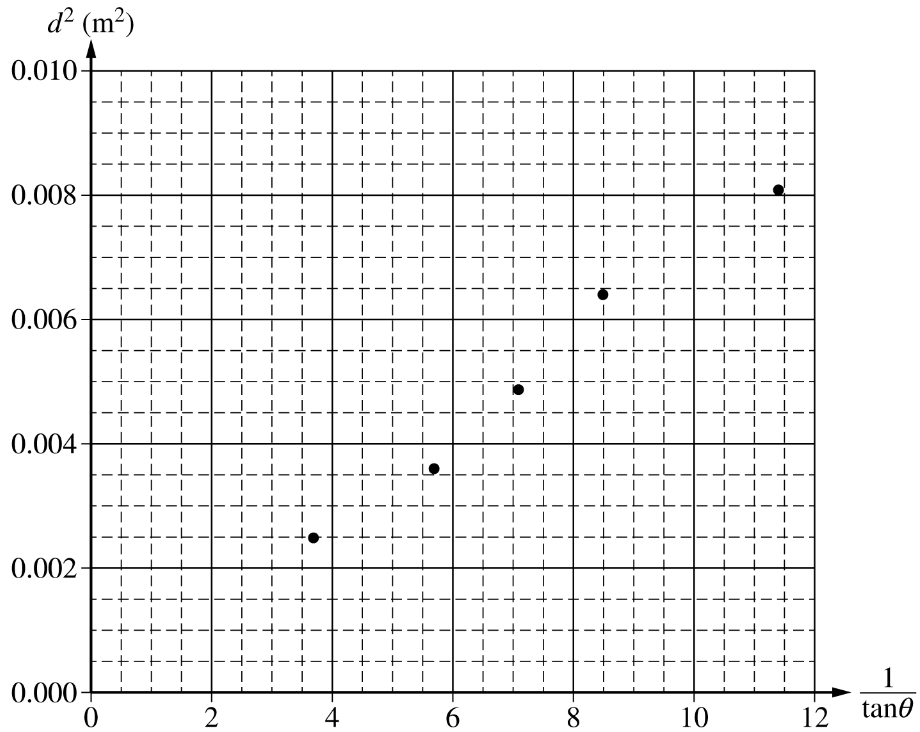
(b) Derive the relationship between the distance d and the angle θ to show that $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg \tan\theta}}$.

(c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8}$ C, $\theta = 12°$, and $d = 0.057$ m. Calculate the expected force of tension exerted on Sphere 2 by the string.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.



i. Draw the best-fit line for the data.

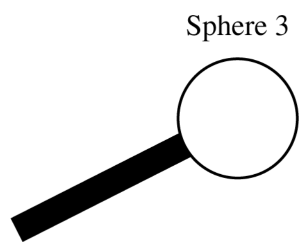
ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $M = 0.0050$ kg and $Q = q = 6.0 \times 10^{-8}$ C.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

i. The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single "+" sign to represent the location of highest concentration of the excess positive charges.



ii. Briefly explain your reasoning for the sketch drawn in part (e)(i).

iii. In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.

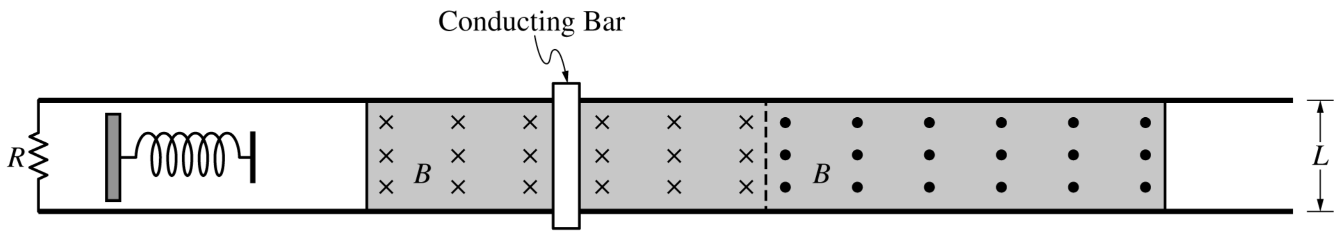
Is θ_2 greater than, less than, or equal to θ_1 ?

_____ $\theta_2 > \theta_1$ _____ $\theta_2 < \theta_1$ _____ $\theta_2 = \theta_1$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.



Top View (at time t_B)

2. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40$ m. A resistor of resistance $R = 0.30\ \Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23$ kg is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1, the bar loses contact with the spring and slides to the right.
- At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50$ T that is directed into the page, as shown.
- At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50$ T but is directed out of the page.
- At time t_4, the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

(a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

GO ON TO THE NEXT PAGE.

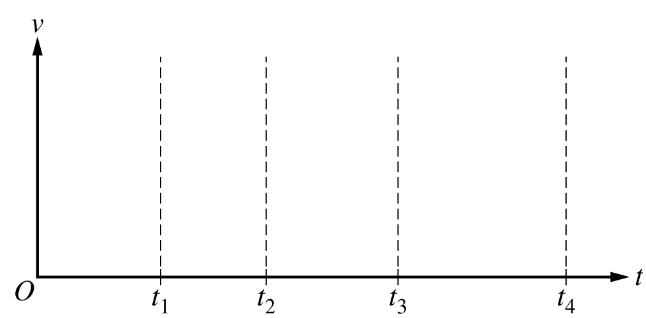
Continue your response to **QUESTION 2** on this page.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

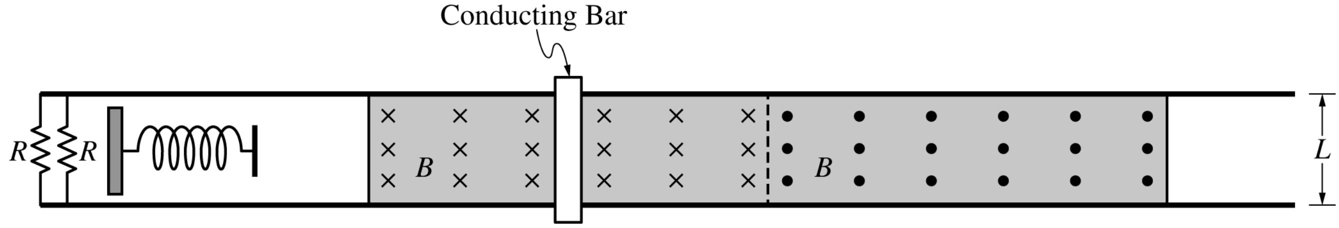
ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.



Top View (at time t_B)

(d) The scenario is repeated but an additional resistor of resistance $R = 0.30\ \Omega$ is connected, as shown.

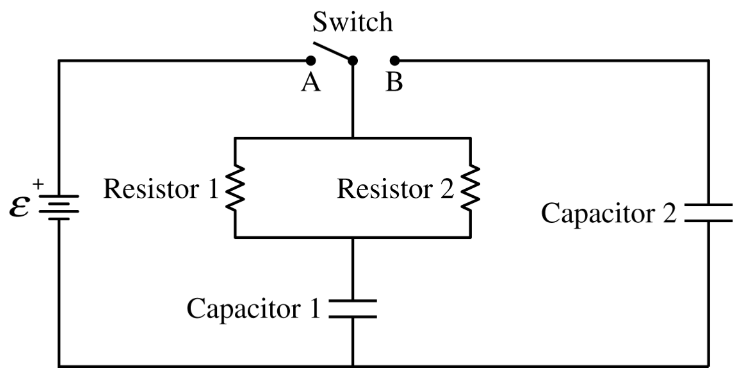
i. Determine the total resistance R_{total} of the closed circuit for the new scenario.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

(e) Describe a modification to m, B, or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

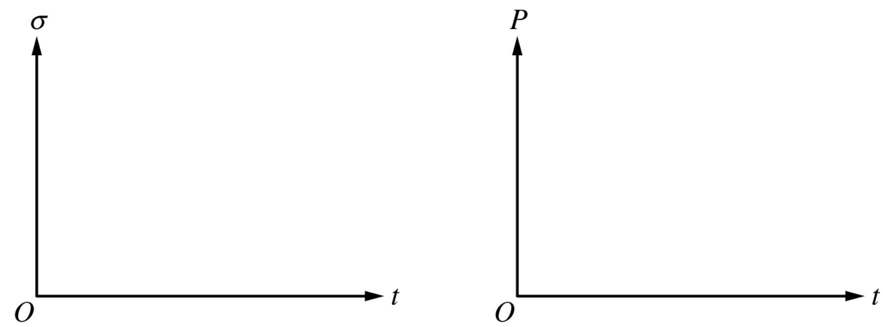
At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

2023

AP® Physics C: Electricity and Magnetism

Free-Response Questions Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^8$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant,	$R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5\ \text{N/m}^2 = 1.0 \times 10^5$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

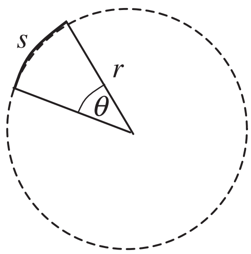
$a^2 + b^2 = c^2$

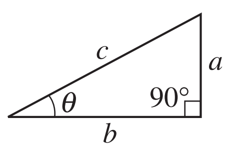
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

Begin your response to **QUESTION 1** on this page.

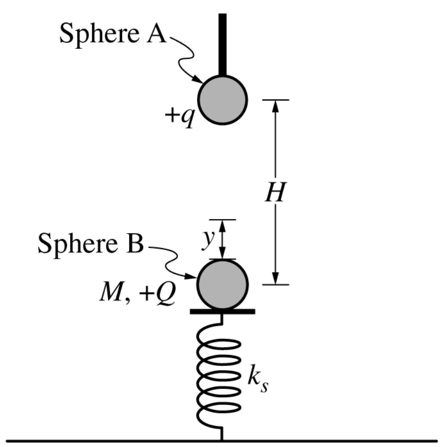
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

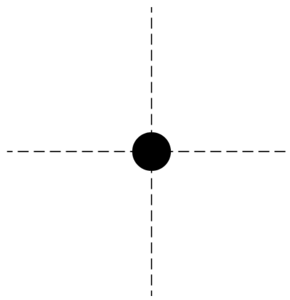
1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

   Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

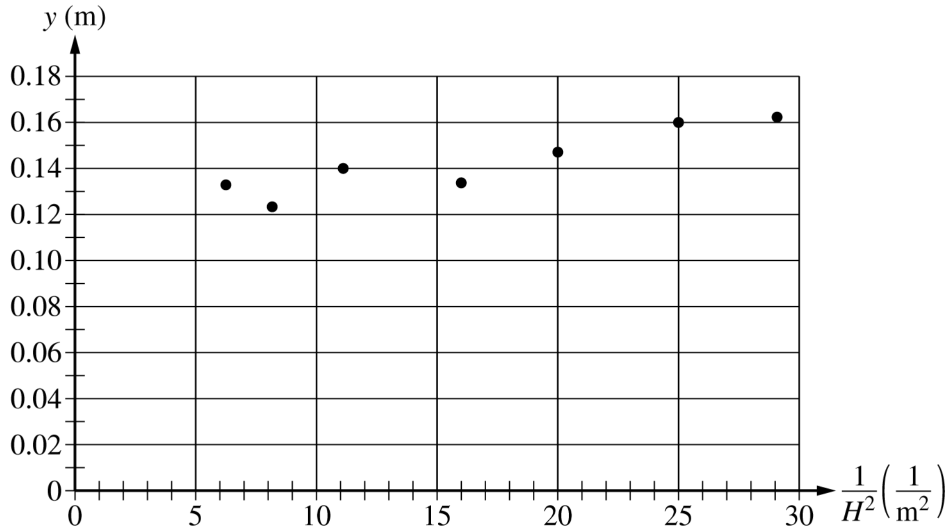


(b) Derive the relationship between y and H to show that $y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m.

iii. Using the best-fit line, calculate an experimental value for the mass of Sphere B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

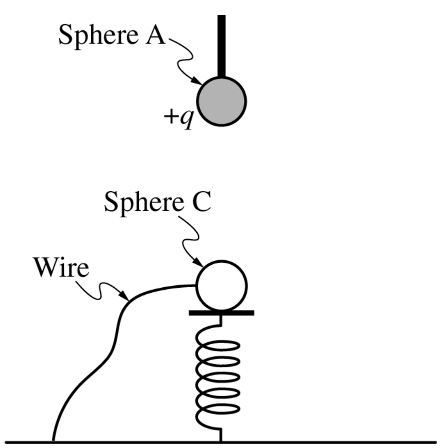
i. In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.

Is y_2 greater than, less than, or equal to y_1 ?

____ $y_2 > y_1$ ____ $y_2 < y_1$ ____ $y_2 = y_1$

Justify your answer.

ii. Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write "does not move."



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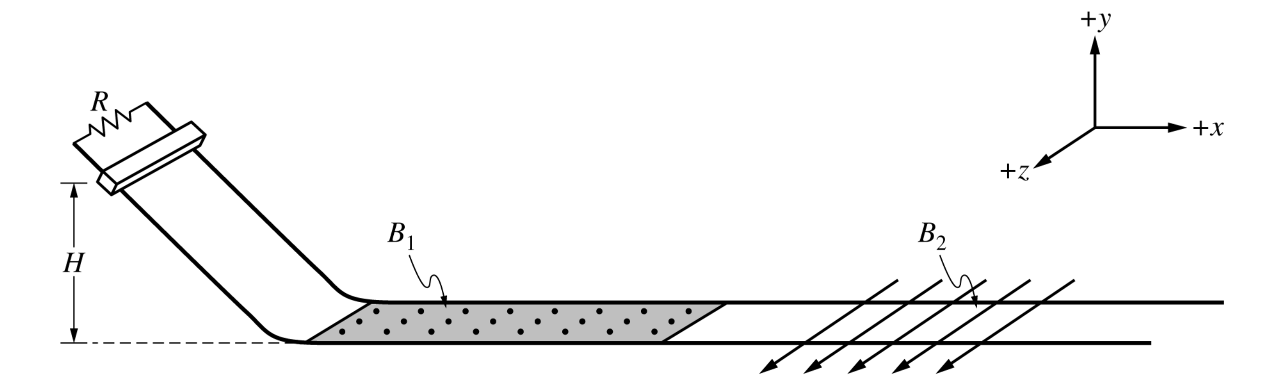
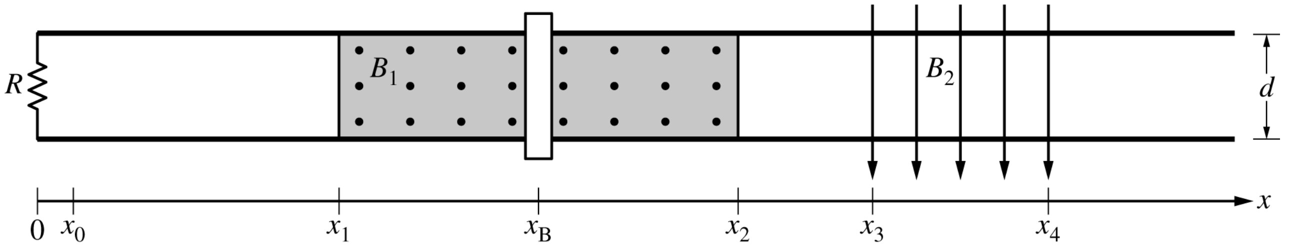
Begin your response to **QUESTION 2** on this page.



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\ \text{m}$. A resistor of resistance $R = 0.20\ \Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\ \text{T}$ that is directed in the $+y$-direction.
- At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\ \text{T}$ that is directed in the $+z$-direction.
- At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

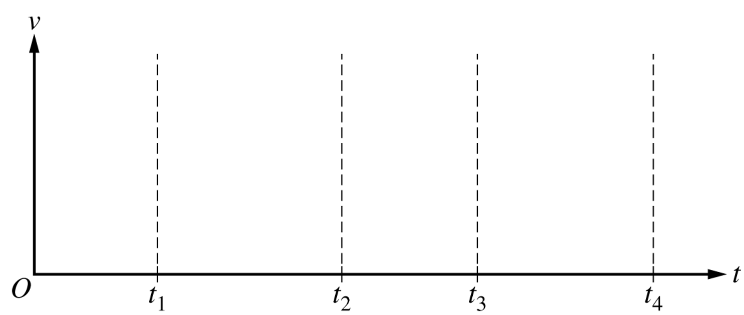
(a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



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Continue your response to **QUESTION 2** on this page.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\ \Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

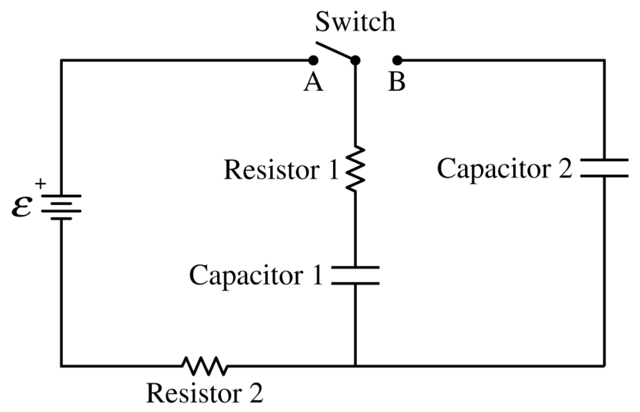
i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

(e) Describe a modification to H, B_1, or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

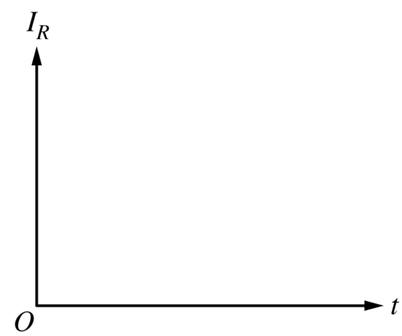
At time $t = 0$, the switch is closed to Position A.

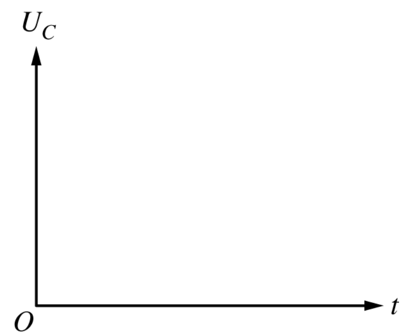
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

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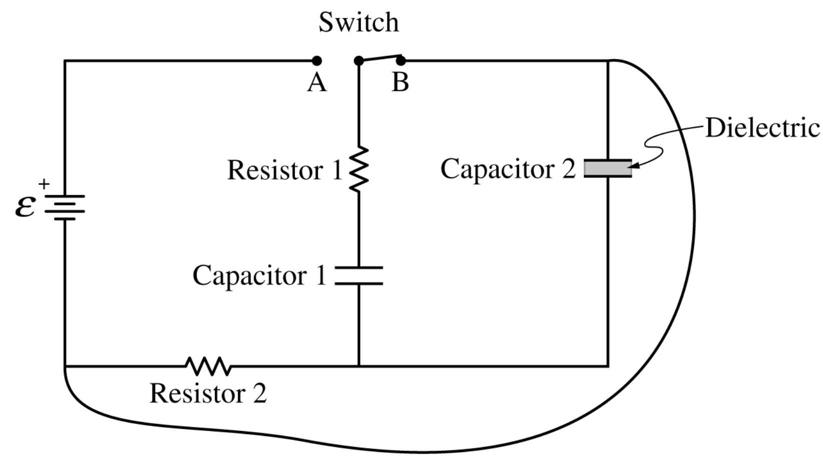
Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

2024

AP® Physics C: Electricity and Magnetism

Free-Response Questions

Set 1

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ $(\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant,	$R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9} (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5} \text{ N/m}^2 = 1.0 \times 10^{5} \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_N\right|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_G\right| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

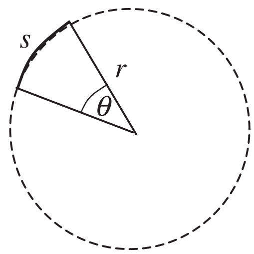
$a^2 + b^2 = c^2$

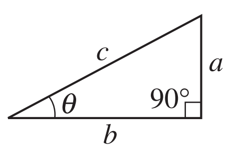
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

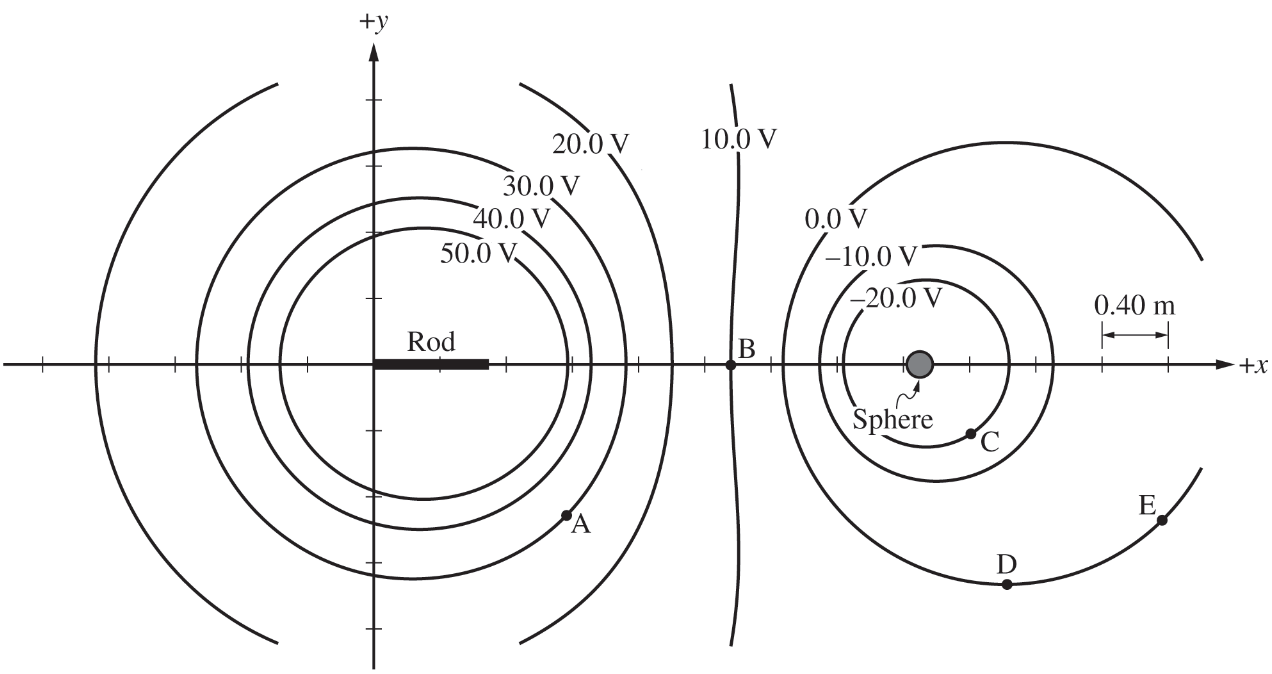


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

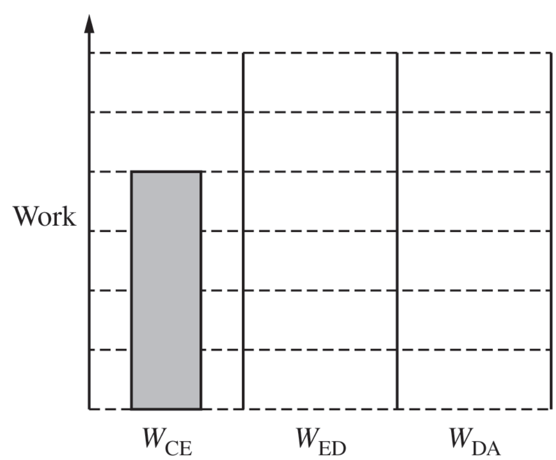


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

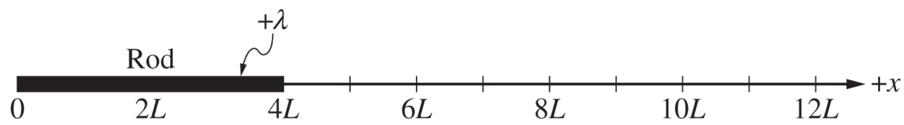


Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the x-component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

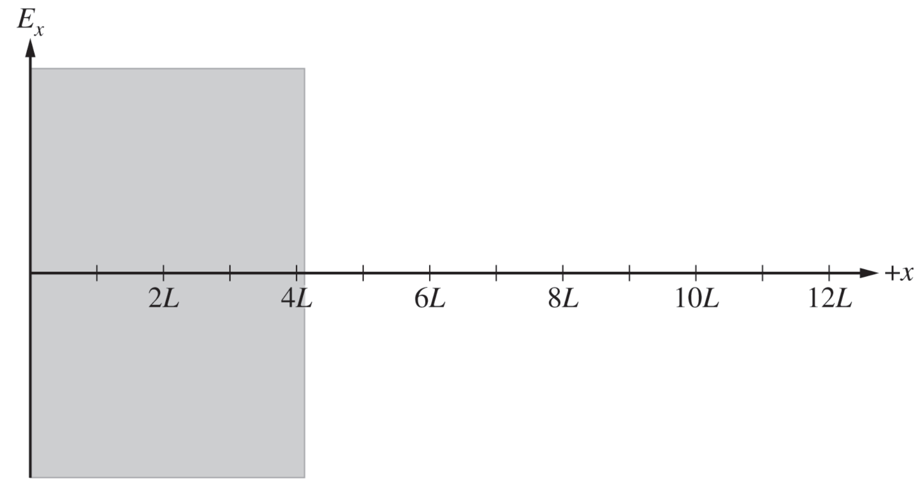


Figure 4

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Begin your response to **QUESTION 2** on this page.

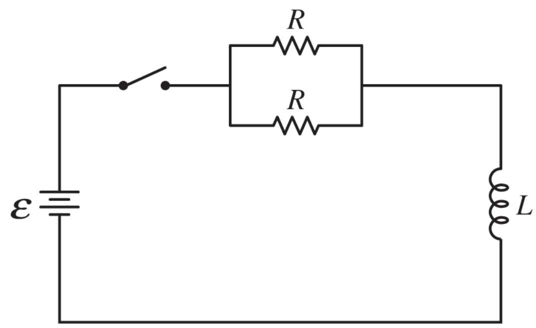


Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.

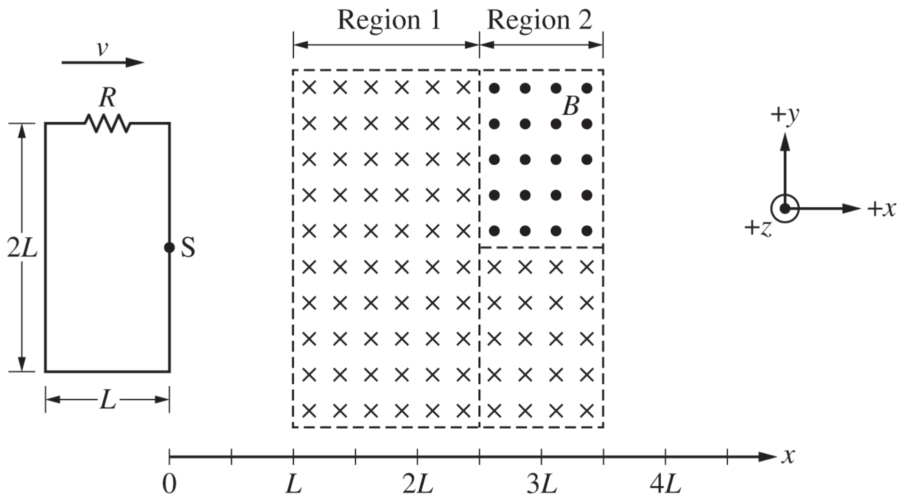


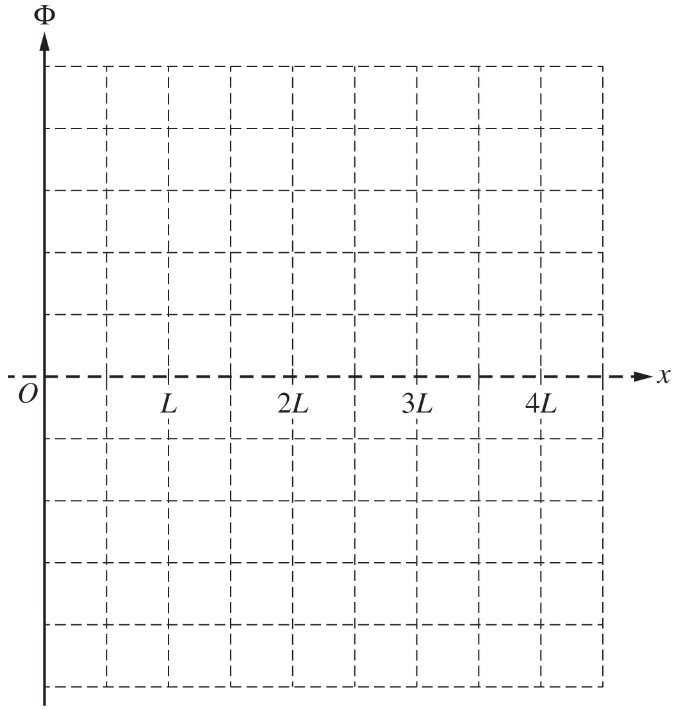
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is $E_{original}$.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new}.

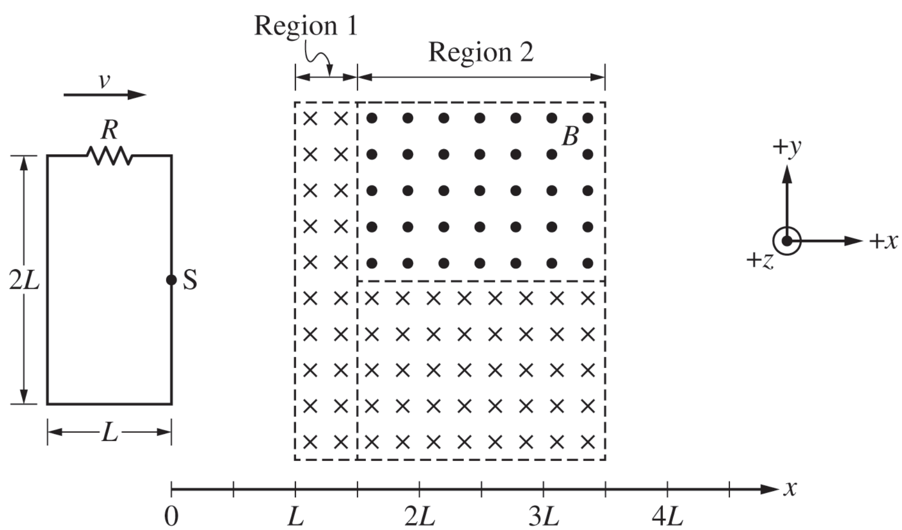


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to $E_{original}$.

______ $E_{new} > E_{original}$ ______ $E_{new} < E_{original}$ ______ $E_{new} = E_{original}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

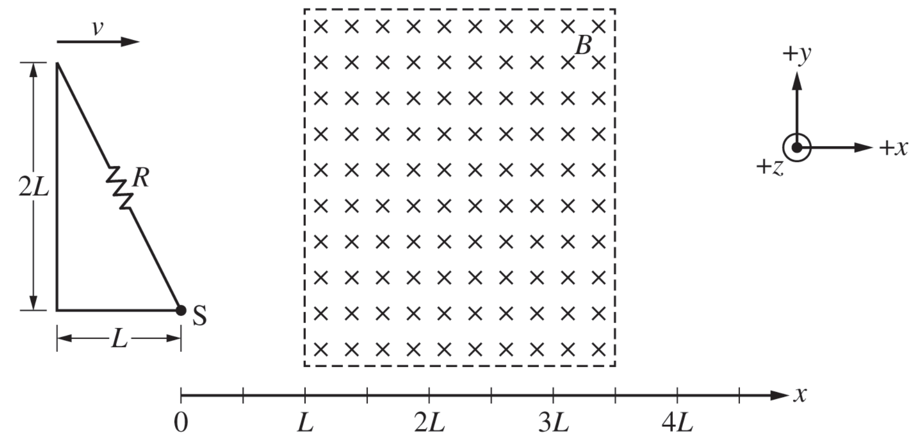
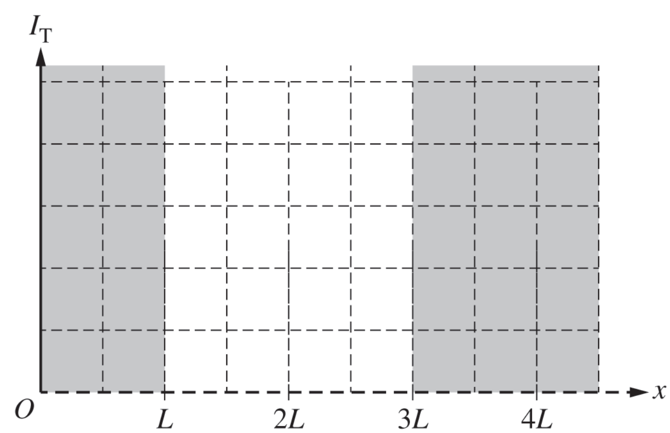


Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the triangular loop as Point S moves from $x = L$ to $x = 3L$.



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STOP

END OF EXAM

2024

AP® Physics C: Electricity and Magnetism

Free-Response Questions

Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant,	$R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$\left|\vec{F}_f\right| \le \mu\left|\vec{F}_N\right|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_G\right| = \frac{Gm_1 m_2}{r^2}$

$U_G = -\frac{Gm_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

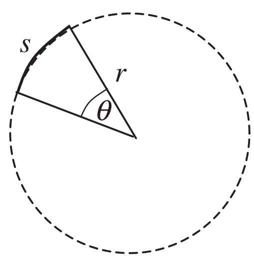
$a^2 + b^2 = c^2$

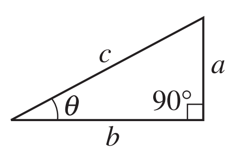
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

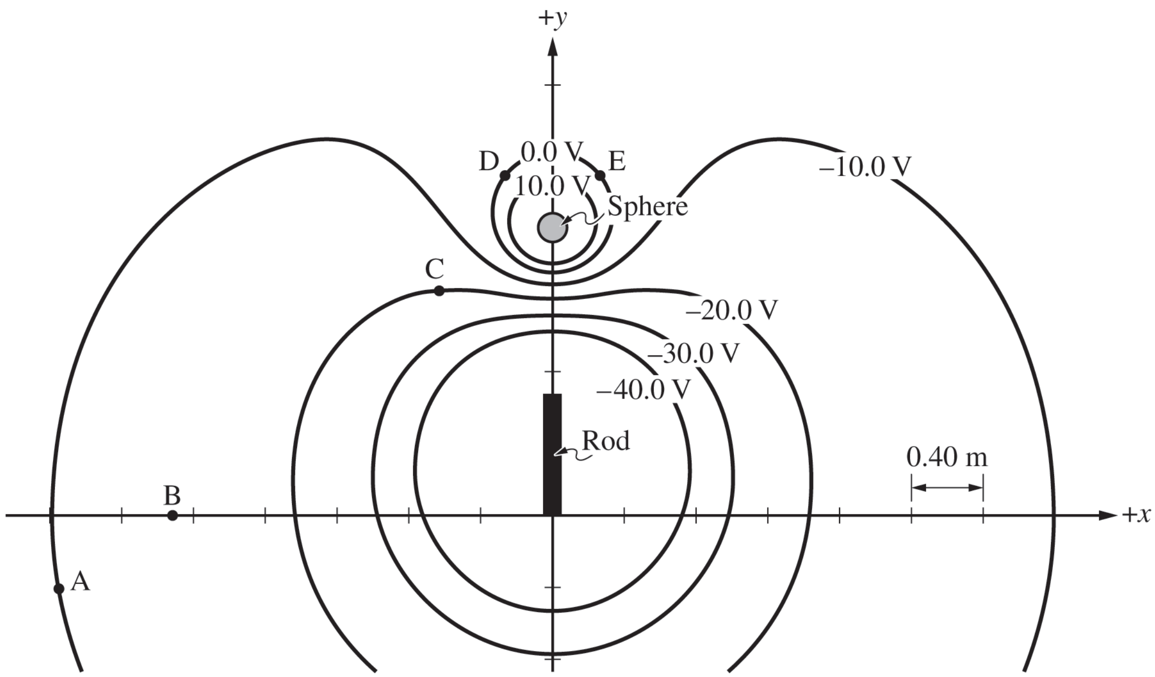


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge +1.0 nC. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

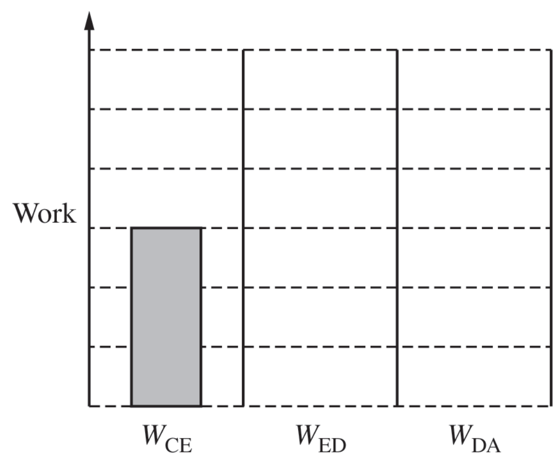


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

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Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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Continue your response to **QUESTION 1** on this page.

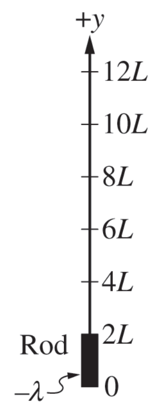

Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

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Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

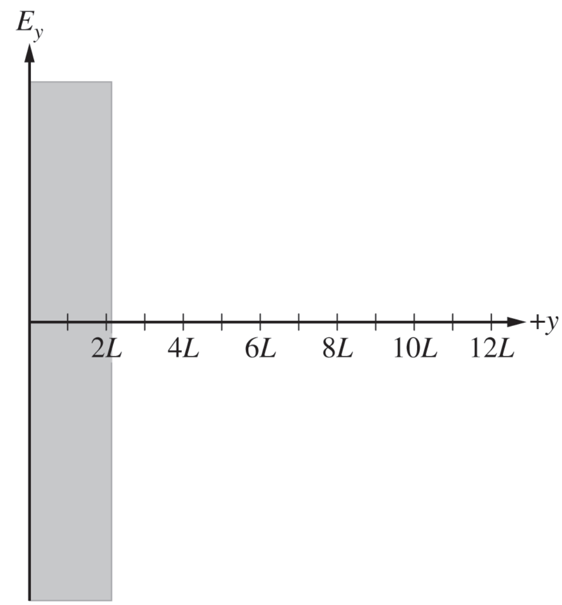

Figure 4

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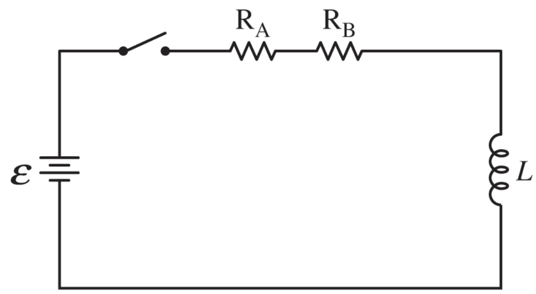

Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

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Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

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Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive,** but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

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Begin your response to **QUESTION 3** on this page.

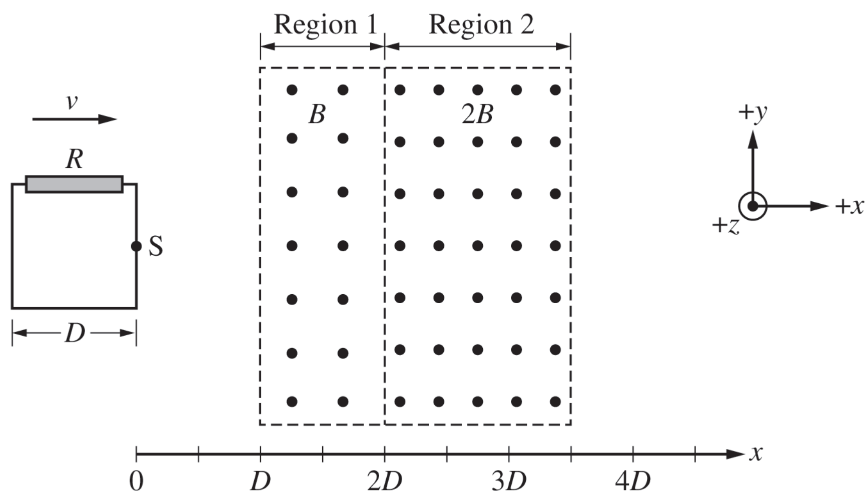

Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

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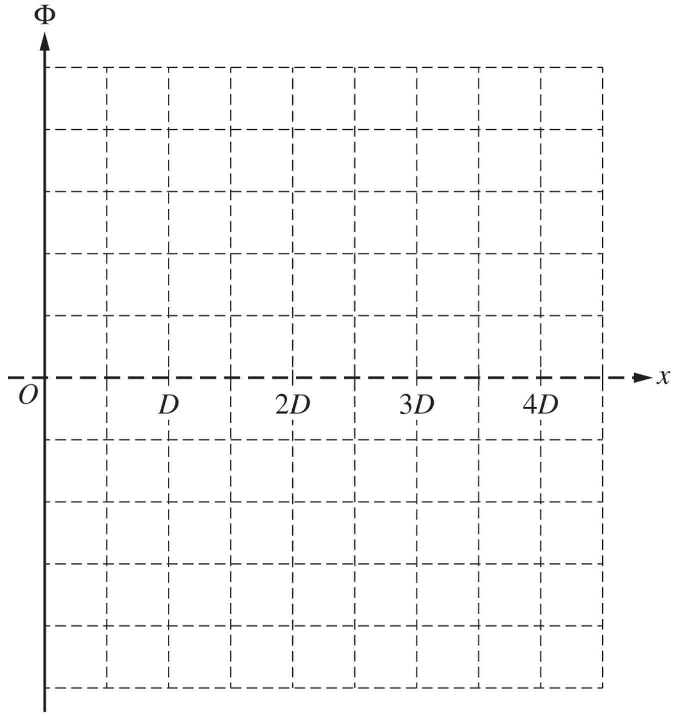
Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the square loop as a function of the position x of Point S from $x = 0$ to $x = 4.5D$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5D$.

i. **Indicate** whether the current I_S that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_____ Clockwise   _____ Counterclockwise   _____ Zero

Briefly **justify** your answer.

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Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_S when Point S reaches $x = 1.5D$. If $I_S = 0$, indicate how the derived expression shows that $I_S = 0$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

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The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is E_{new}.

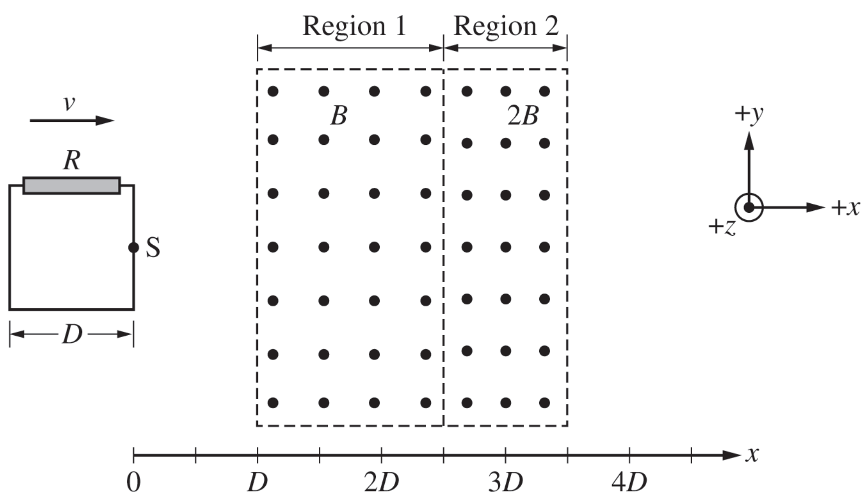

Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

______ $E_{\text{new}} > E_{\text{original}}$ ______ $E_{\text{new}} < E_{\text{original}}$ ______ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

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The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field of magnitude B that is directed in the $+z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length D and height D. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the upper-leading corner of the loop.

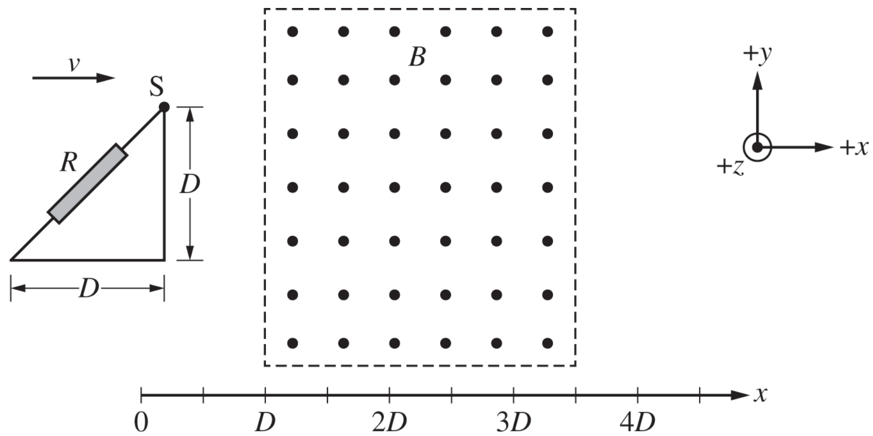
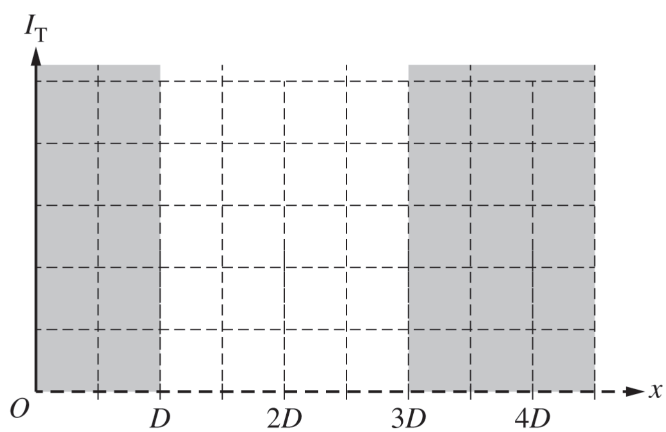


Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the loop as Point S moves from $x = D$ to $x = 3D$.



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END OF EXAM