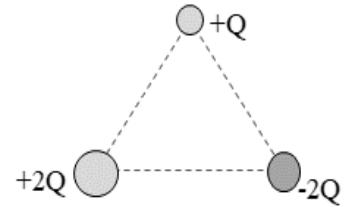







**Part 1: Multiple Choice** - Choose the answers that best answers the questions below. If an exact answer is not present, chose the closest available answer. (4 points each)

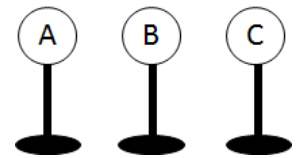
1. Three point charges, of charges  $+Q$ ,  $-2Q$ , and  $+2Q$ , are placed equidistant as shown on the right. Which vector best describes the net direction of the electric force acting on the  $+Q$  charge?



- A)  B)  C)   
 D)  E) 

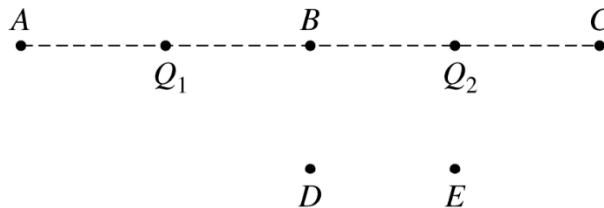
2. If the only force acting on electron is due to a uniform electric field, the electron moves with constant:  
 A) acceleration in a direction opposite to that of the field.  
 B) acceleration in the direction of the field.  
 C) acceleration in a direction perpendicular to that of the field.  
 D) speed in a direction opposite to that of the field.  
 E) speed in the direction of the field.

3. Three identical conducting spheres are mounted on insulating stands as shown. Sphere A and C have equal chargers of  $+Q$  and are seperated by a fixed distance. They repel each other with an electrostatic force of magnitude,  $F$ . Sphere B is initially uncharged. Sphere B is first touched to sphere A (then moved away from A), then touched to sphere C, and then removed from the setup completely. If the charge distribution on each sphere is assumed to always be spherical, what would be the new magnitude of the electrostatic force between spheres A and C?



- A) 0                      B)  $\frac{3F}{8}$                       C)  $\frac{F}{2}$                       D)  $\frac{F}{16}$                       E)  $\frac{F}{4}$

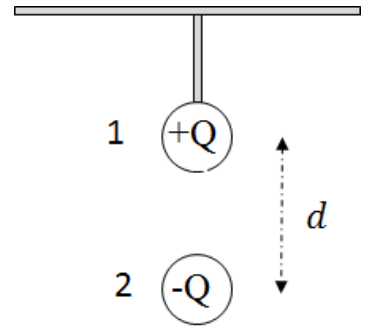
4. Two point charges of  $Q_1 = +8\mu C$  and  $Q_2 = +8\mu C$  are situated as shown below.



At which labeled point above is the electric field strength the greatest?

- A) A                      B) B                      C) C                      D) D                      E) E

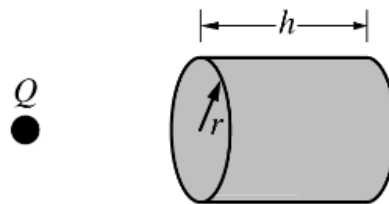
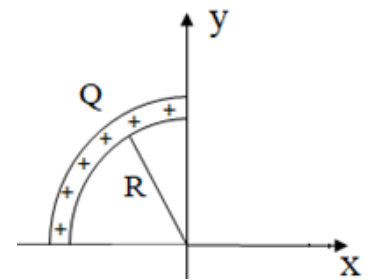
5. In the diagram shown, sphere 2 remains at rest in the air below sphere 1, which is attached to insulating stand. Sphere is distance of  $d$  below sphere 1. The sphere have equal masses  $M$ , and opposite charges of  $+Q$  and  $-Q$ . Which the of following gives the magnitude of  $Q$ ?



- A)  $2d\sqrt{\pi\epsilon_0 Mg}$       B)  $2\sqrt{\frac{\pi\epsilon_0 Mg}{d}}$       C)  $\sqrt{\frac{Mg}{4\pi\epsilon_0 d}}$   
 D)  $\frac{4\pi\epsilon_0 Mg}{d}$       E)  $\frac{2}{d}\sqrt{2\pi\epsilon_0 Mg}$

6. A circle of charge is to be placed centered about the origin. However, due to the government shutdown, only a quarter circle of charge  $+Q$  is constructed in the second quadrant of the Cartesian plane. Which of the following integrals would, when evaluated, give the x-component of the electric field at the origin due to the quarter-circle of charge?

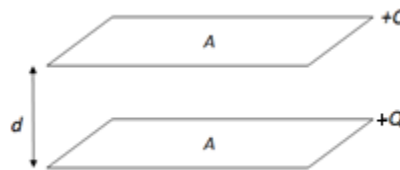
- A)  $\int_{\pi/2}^{\pi} \frac{kQ}{2\pi R^2} \cos \theta d\theta$       B)  $\int_{\pi/2}^{\pi} \frac{2kQ}{\pi R^2} \cos \theta d\theta$   
 C)  $\int_{\pi/2}^{\pi} \frac{kQ}{\pi R} \cos \theta d\theta$       D)  $\int_{\pi/2}^{\pi} \frac{2kQ}{\pi R} \sin \theta d\theta$   
 E)  $\int_{\pi/2}^{\pi} \frac{kQ}{\pi R^2} \sin \theta d\theta$



7. A closed cylindrical shell of volume  $\pi r^2 h$  is placed close to an object with a charge of  $Q$ , as shown above. There are no other charged objects nearby. The electric flux through the closed cylindrical shell is:

- A)  $\frac{Q}{\epsilon_0}$       B)  $\pi r^2 Q$       C)  $2\pi r h$       D) 0      E)  $2\pi r^2 Q + 2\pi r h Q$

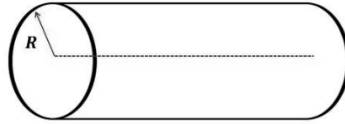
8. Two parallel plates has the same charge  $+Q$ ,  $A$ . The plates are separated by a distance  $d$  as shown.



What is the magnitude of the electric field in the region between the plates?

- A)  $\frac{Q}{A\epsilon_0}$       B)  $\frac{2Q}{A\epsilon_0}$       C)  $\frac{Q}{d\epsilon_0}$       D)  $\frac{Q}{2A\epsilon_0}$       E) 0

9. A very long nonconducting cylinder has a radius of  $R$  and a volume charge density of  $\rho$ . Which of the following is the correct setup for Gauss's law to determine  $E$ , the magnitude of the electric field due to the cylinder a distance  $r$  from the center, where  $r > R$ .



A)  $E(2\pi R) = \frac{\rho\pi r^2}{\epsilon_0}$       B)  $E(2\pi r) = \frac{\rho\pi r^2}{\epsilon_0}$       C)  $E(4\pi r^2) = \frac{\rho\pi R^2}{\epsilon_0}$

D)  $E(4\pi r^2) = \frac{\rho\pi r^2}{\epsilon_0}$       E)  $E(2\pi r) = \frac{\rho\pi R^2}{\epsilon_0}$

10. Appreciate that there is a line of charge of length  $L$  and charge  $Q$ . Point  $P$  is somewhere near the middle of the line of charge, a distance of  $d$  away, where  $L \gg r$ . Which of the following is a valid application of Gauss's law to find the magnitude of the electric field at point  $P$ ?

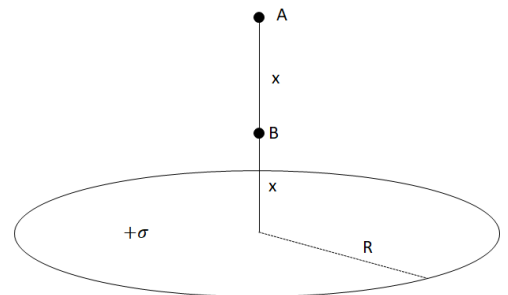
A)  $E(2\pi r x) = \frac{Qx}{L\epsilon_0}$       B)  $E(2\pi r^2) = \frac{Q}{L\epsilon_0}$       C)  $E(\pi r^2) = \frac{Q}{L\epsilon_0}$

D)  $E(2\pi r L) = \frac{Q}{L\epsilon_0}$       E)  $E(2\pi r x) = \frac{QL}{x\epsilon_0}$

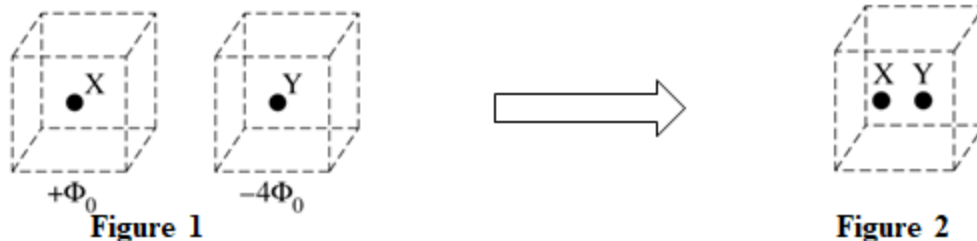
11. A long cylinder (radius = 3.0 cm) is filled with a nonconducting material which carries a uniform charge density of  $1.3 \mu\text{C}/\text{m}^3$ . Determine the electric flux through a spherical surface (radius = 2.0 cm) which has a point on the axis of the cylinder as its center.

A)  $5.7 \text{ Nm}^2/\text{C}$       B)  $4.9 \text{ Nm}^2/\text{C}$       C)  $6.4 \text{ Nm}^2/\text{C}$       D)  $7.2 \text{ Nm}^2/\text{C}$       E)  $15 \text{ Nm}^2/\text{C}$

12. A flat charged disk lies in the  $xy$  plane as shown. Points  $A$  and  $B$  are points directly above the center of the disk as shown. Point  $A$  is twice as far from point  $B$  from the disk, and the disk has a radius such that  $R \gg x$ . Which of the following gives the value of the ratio:  $\frac{\text{Electric field magnitude at } A}{\text{Electric field magnitude at } B}$ ?



A)  $\frac{1}{2}$       B) 2      C)  $\frac{1}{4}$       D) 4      E) 1



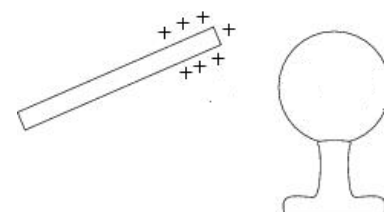
13. Figure 1 above shows two charged spherical conductors, X and Y, which are equal in size. When each conductor is isolated and surrounded by a closed cubical surface, the total electric flux through the surfaces is  $+\Phi_0$  for conductor X and  $-4\Phi_0$  for conductor Y. Conductor Y is brought into contact with conductor X and then separated, as shown in Figure 2. If the separation is small so that both conductors are inside the same closed cubical surface, as shown above, what is the total electric flux through the surface?

- A)  $-\frac{5}{2}\Phi_0$       B)  $-\frac{3}{2}\Phi_0$       C)  $-3\Phi_0$       D)  $\frac{5}{2}\Phi_0$       E) 0

14. A long non-conducting cylinder of radius R has a uniform volume charge density. At the lateral surface of the cylinder, the electric field strength is E. A second cylinder of radius 2R is created with the same volume charge density. What is the electric field strength at the lateral surface of the second cylinder?

- A) 4E      B) E/2      C) E      D) E/4      E) 2E

15. Congratulations on making through the multiple choice. Here's an easy question. A positively charged glass rod is brought close to, but does not touch, a conducting sphere. Which of the following describes the resulting charge on the sphere?

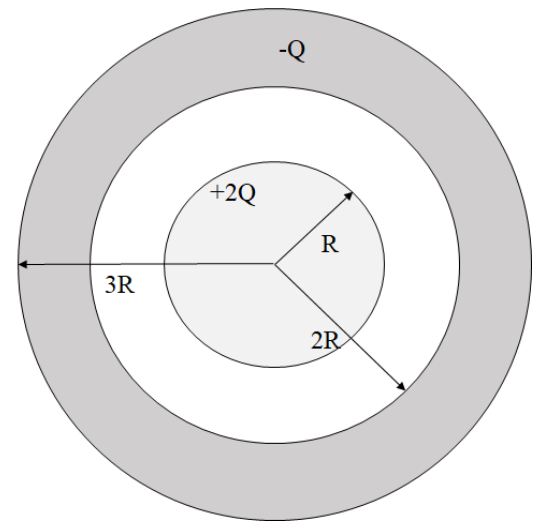


- A) The sphere is positively charged.  
 B) The sphere is negatively charged.  
 C) The charged is uncharged, but the sphere is polarized, with positive charge moving right.  
 D) The charged is uncharged, but the sphere is polarized, with positive charge moving left.  
 E) All of the above.

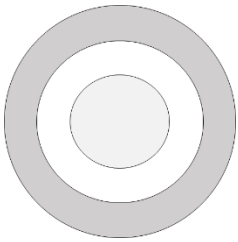
Part 2: Free Response. **You must show all steps** required to arrive at the correct answer for the following problems, including any diagrams. **All answers must be given with correct units.**

16. (27 points) A uniform solid conducting sphere has a charge of  $+Q$  and radius  $R$ . As is tradition, it is surrounded by a spherical shell of inner radius  $2R$  and  $3R$ . The outer shell is a non-conductor with a charge of  $-Q$  distributed uniformly through its volume.

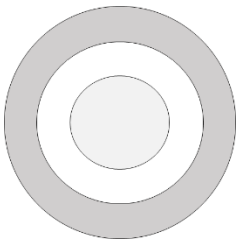
a) Determine a function for the electric field,  $E(r)$ , for the following regions. For each region, clearly draw and identify the gaussian surface you are using.



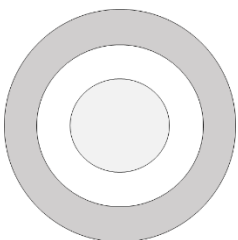
i.  $r < R$



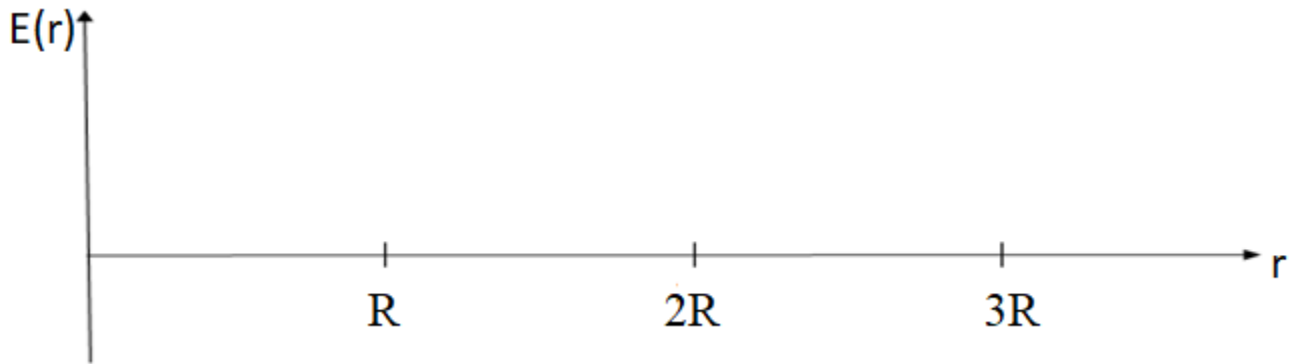
ii.  $2R < r < 3R$



iii.  $r > 3R$

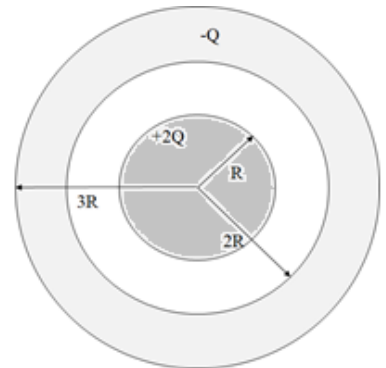


iv. Sketch the field  $E(r)$ , for the field as a function of distance,  $r$ , from the center of the sphere.



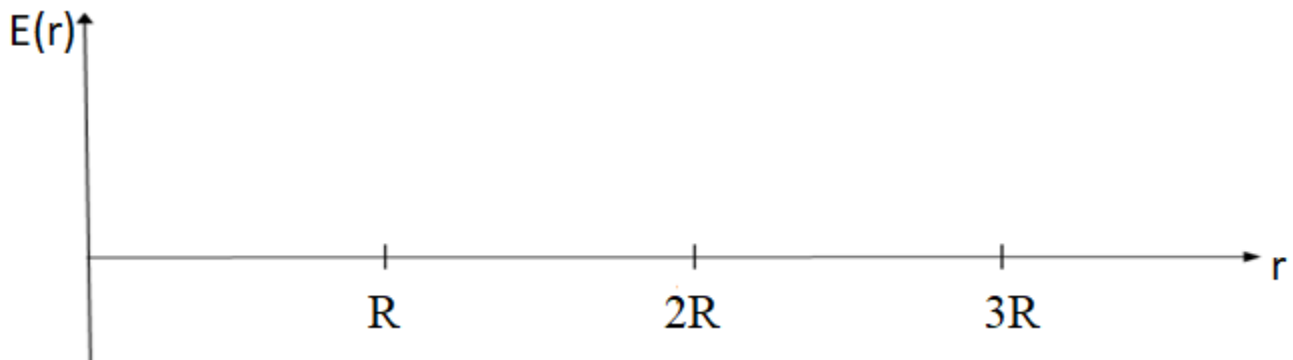
b) It's rewind time. The inner sphere is replaced with a solid, uniform conducting material, with the same charge  $+2Q$ , and the outer shell is now replaced with a conducting shell of the same inner and outer radii and still carries a total charge of  $-Q$ .

i. Determine an expression for the charge density on the inner surface of the outer shell.



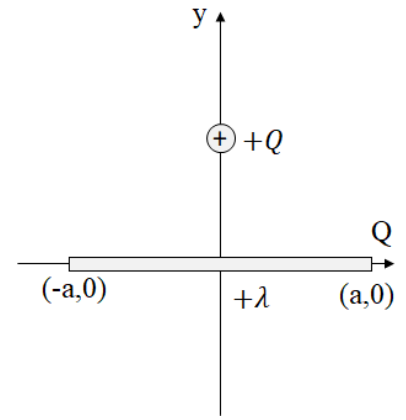
ii. Determine an expression for the charge density on the inner surface of the outer shell.

iii. Graph the field vs. distance for this case.



16. (15 points) A charged point of charge  $Q$  is placed at the point  $(0,b)$ . The charge is affected by a charged rod of total charge  $+Q$  that is placed along the  $x$ -axis from  $(-a,0)$  to  $(a,0)$  as shown.

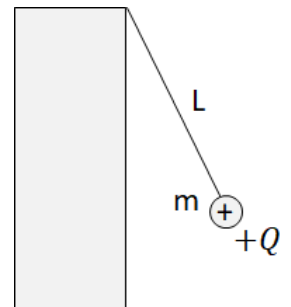
a) Write a differential equation that could be used to find the force on the point charge due to the rod. Ensure to include bounds in the integral and put the function to be integrated in simplest form.



b) The charge is allowed to move. Assuming there is no gravity, describe the result motion of the charge in terms of its speed and acceleration from its release to a very long time after it's released.

c) The charge of mass  $Q$  is attached to one end of a string of length  $L$ . The charge is a small point of mass  $M$ . The other end is fixed to a wall. The wall causes a constant, uniform electric field of magnetic field  $E$ .

i. What is the direction of the electric field created by the wall? Justify your answer.



ii. Determine an expression for the angle the string makes with the wall. Show your work, including the relevant free body diagram.

**Bonus:** A spherical cloud of charge of radius  $R$  contains a total charge  $+Q$  with nonuniform volume charge density that varies according to:

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \text{ for } r \leq R$$

Determine the electric field as a function of  $r$  within the cloud.

