AP Physics C: Mechanics
Unit 5&6 Practice Exercises

Directions: Show the steps required to arrive at the answer (if applicable). Use g = 9.80 m/s^2 and neglect air resistance unless otherwise stated. Work out the problems on separate page.

5.1 – Rotational Kinematics

1. Calculate the angular velocity of the Earth.

2. A record player is initially at rest when it is set into motion with a constant angular acceleration α. What is the angular velocity of the record player after it has made one complete revolution?

3. A centrifuge rotates with an angular velocity given by $\omega(t) = 20 - 10t^2$. Calculate the average angular acceleration from t = 0 s to t = 4 s.

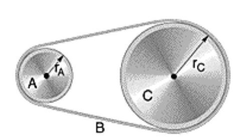
4. A circular saw blade 0.4 m in diameter starts from rest and accelerates with constant angular acceleration to an angular velocity of 200 rad/s in 25 s. Find the angular acceleration and the angle through which the blade has turned.

5. A frog rides on a unicycle. The unicycle wheel accelerates uniformly from a speed of 5.0 rev/s with a rate of acceleration of 5 rad/s^2. The wheel makes 25 complete revolutions as it accelerates. Find the final speed of the wheel.

6. A spinning thing is initially spinning (at time t=0) at an angular velocity 6 radians per second with no angular displacement an angular acceleration given by $\alpha(t) = 18t^2$. Find the number of complete revolutions made by the object at time t = 10 s.

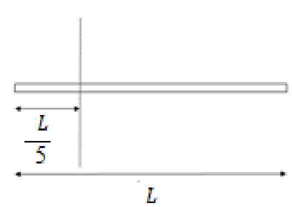
7. An astronaut is being tested in a centrifuge. The centrifuge has a radius of 10 m and, in starting, rotates according to $\theta = 0.3t^2$, where t is in seconds, and θ is in radians. When t = 5.0 s, what are the magnitudes of the astronaut's
   a) angular and linear velocity?
   b) tangential acceleration and radial acceleration?

8. In the figure shown, wheel *A* of radius r_A = 0.1 m is coupled by belt *B* to wheel *C* of radius r_C = 0.2 m. The angular speed of wheel *A* is increased from rest at a constant rate of 2.0 rad/s^2. Find the time for wheel *C* to reach a rotational speed of 100 rad/s, assuming the belt does not slip.

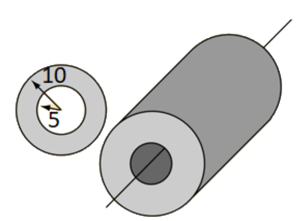


5.2 – Moment of Inertia

1. A righteous rod has a length L and mass M. Using integral calculus, determine an expression for the moment of inertia about an axis that is a distance of L/5 from the rod's left end, as shown.

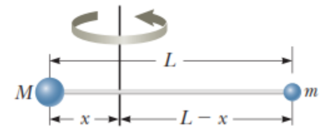


2. A cylinder has an outer radius of 10 m and an inner radius of 5 m as shown. Use integral calculus to derive an expression for the moment of inertia of the cylinder.



3. A hollow sphere of mass M and radius R rotates about an axis tangent to the sphere. Find the moment of inertia.

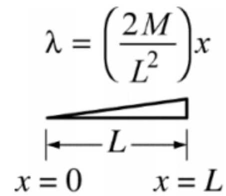
4. Two balls with masses M and m are connected by a rigid rod of length L and negligible mass as shown. For an axis perpendicular to the rod:



a) show that the system has the minimum moment of inertia when the axis passes through the center of mass.

b) Show that this moment of inertia is $L^2 mM/(m+M)$.

5. A triangular rod has length L, mass M, and nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod, a shown.



a) Show that the rotational inertia of the rod about its left end is $ML^2/2$.

b) The thin hoop below in Figure 1 has mass M, radius L, and rotational inertia around its center ML^2. Three rods identical to the rod from part a) are now fastened to thing hoops, as shown in Figure 2.

Derive an expression for the rotational inertia of the hoop-rods system about the center of the hoop.

<u>5.3 – Torque</u>

1. A car traveling on a flat (unbanked), circular track accelerates uniformly from rest with a tangential acceeration of a. The car makes it one-quarter of the way around the circle before it skids off the track. From these data, determine the coefficient of static friction between the car and the track.

2. In a manufacturing process, a large, cylindrical roller is used to flatten material fed beneath it. The diameter of the roller is 1.00 m, and, while being driven into rotation around a fixed axis, its angular position is given by:

$$\theta(t) = 2.5t^2 - 6t^3$$

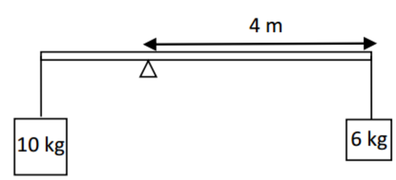
a) Find the maximum angular speed of the roller.

b) What is the maximum tangential speed of a point on the rim of the roller?

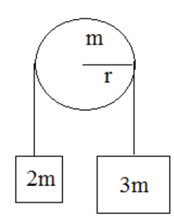
c) At what time t should the driving force be removed from the roller so that the roller does not reverse its direction of rotation?

d) Through how many rotations has the roller turned between $t = 0$ and the time found in part (c)?

3. Two masses of mass 10 and 6 kg are hung from massless strings at the end of a light rod. The rod itself is virtually weightless. A pivot is placed off center and the system is free to rotate. The system is released from rest. Determine the initial acceleration of the system.



4. A pulley with non-negligible friction has two weights of objects 2m and 3m as shown. The pulley with a wheel with moment of inertia I=$\frac{1}{2}mr^2$.

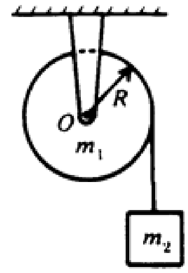


a) Determine an expression for the acceleration of the masses.

b) If a frictionless pulley is used instead, how would your answer compare?

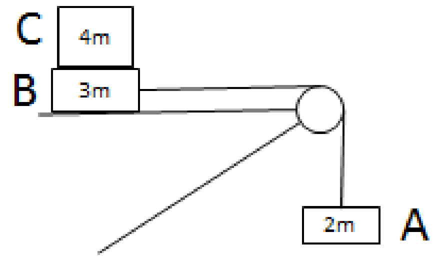
5. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is I = ½m_1R^2. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time t = 0. In terms of m_1, m_2, R. and g, determine each of the following.

a) The acceleration of the block.
b) The tension in the cord.
c) The angular momentum of the disk as a function of time t.



6. The horizontal surface on which block B moves is frictionless. The coefficient of kinetic friction between blocks B and C is μ. The pulley has moment of inertia I = $3MR^2$ and a radius of R. Block A is released from rest.

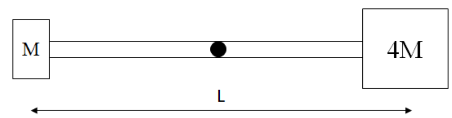
a) Find the downward acceleration of block A.
b) Find the tension between A and the pulley.
c) Find the tension between B and the pulley.
d) Find the acceleration of block C.



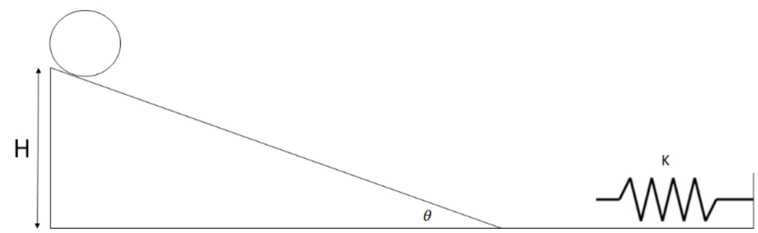
5.4 – Work & Energy in Rotation

1. A rod of mass M and length L is pivoted about its center (moment for inertia for a rod pivoted at center: $I = \frac{mL^2}{12}$). Two mass are attached at each end of the rod such that each mass's center is at the end of the rod. The system is released from rest and allowed to rotate.

a) What is the angular acceleration of the system the moment it's released from the position shown?
b) The rod rotates and ends up in the vertical position as shown, with the 4m mass being on the bottom. Calculate the angular velocity of the system at this point.

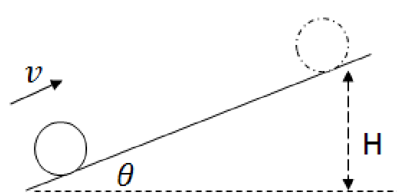


2. A solid baseball ($I = \frac{2}{5}mr^2$) has a mass of m and a radius of r and is placed at the top of an incline of height H that is incline at an angle θ above the horizontal. Assume the ball rolls without slipping for the duration of this problem. Just for fun, an ideal spring is placed at the bottom of the incline with an elastic constant of K.

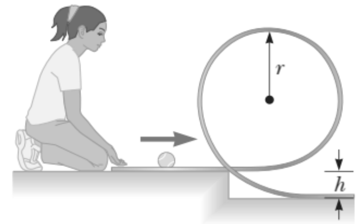


a) Determine an expression for the minimum coefficient of friction on the incline to prevent slipping.
b) Determine an expression for the linear speed of the ball's center of mass at the bottom of the incline.
c) How would your answer to b) change if:
   i. a hollow ball was used? Justify your answer.
   ii. the ball rolls and slips down the incline. Justify your answer.
d) Determine an expression for the maximum distance that the spring is compressed.

3. A ball of mass M and radius R with $I = \frac{2}{3}MR^2$ moves up an incline as shown. The speed of the ring is v as it enters the incline. Determine an expression for the force of friction on the incline.



4. A tennis ball is a hollow sphere with a thin wall. It is set rolling without slipping at 4.00 m/s on a horizontal section of a track as shown. It rolls around the inside of a vertical circular loop of radius r =45.0 cm. As the ball nears the bottom of the loop, the shape of the track deviates from a perfect circle so that the ball leaves the track at a point h 5 20.0 cm below the horizontal section.



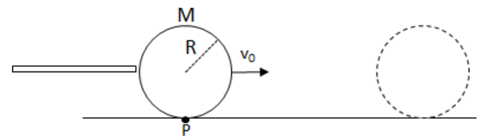
a) Find the ball's speed at the top of the loop.

b) Demonstrate that the ball will not fall from the track at the top of the loop.

c) Find the ball's speed as it leaves the track at the bottom.

d)Suppose that static friction between ball and track were negligible so that the ball slid instead of rolling. Would its speed then be higher, lower, or the same at the top other loop? Explain.

5. A billiard ball has mass M, radius R, and is a solid sphere (I=$\frac{2}{5}mr^2$). The ball is struck by a cue stick along a horizontal line through the ball's center of mass so that the ball initially slides with a velocity v_0 as shown. As the ball moves across the rough billiard table (with a coefficient of sliding friction μ), its motion gradually changes from pure translation through rolling with slipping to rolling without slipping.



a) Develop an expression for the linear velocity v of the center of the ball as a function of time while it is rolling with slipping.

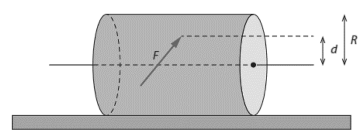
b) Develop an expression for the angular velocity ω of the ball as a function of time while it is rolling with slipping.

c) Determine the time at which the ball begins to roll without slipping.

d) Describe what is meant by rolling without slipping.

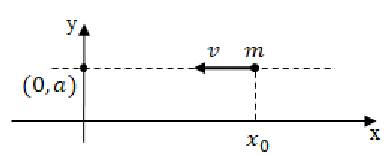
e) When the ball is struck it acquires an angular momentum about the fixed point P on the surface of the table. During the subsequent motion the angular momentum about point P remains constant despite the frictional force. Explain why this is so.

6. A horizontal force F is applied to the surface of a cylinder of mass M and radius R. The force is applied a vertical distance d above its centre of mass, as shown below. Determine d as a fraction of R such that the cylinder rolls without slipping. The moment of inertia of a cylinder about its central axis is $\frac{1}{2}MR^2$.

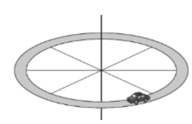


5.5 – Angular Momentum

1. A particle of mass m moves with a constant speed v along the dashed line $y = a$. When the x-coordinate of the particle is x_0, what is the magnitude of the angular momentum of the particle with respect to the origin of the system?

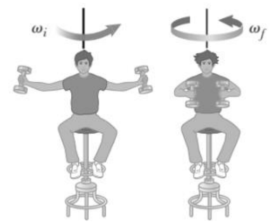


2. A battery-driven toy car of mass 0.18 kg is placed on a circular track that is part of a horizontal ring with radius 0.50 m and moment of inertia 0.20 kgm^2 relative to its vertical axis. The ring can rotate about this axis without friction. The car is started and its speed is measured to be 0.80 m s−1 relative to the ground. Calculate the angular speed of the ring.



3. A star of mass M and radius R explodes radially and symmetrically. The star is left with a mass M/10 and R/50. Calculate the ratio of the star's final angular velocity to its initial angular velocity.

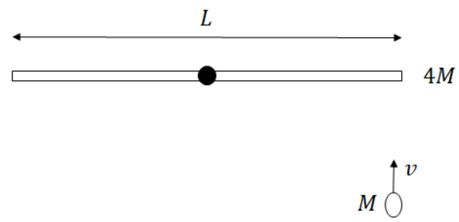
4. Derek sits on a freely rotating stool holding two dumbbells, each of mass 4.00 kg. When his arms are extended horizontally, the dumbbells are 1.00 m from the axis of rotation and the student rotates with an angular speed of 0.750 rad/s. The moment of inertia of the student plus stool is 4.00 kg·m^2. The student pulls the dumbbells inward horizontally to a position 0.20 m from the rotation axis



a) Find the new angular speed of Derek.

b) Find the kinetic energy of the rotating system before and after he pulls the dumbbells inward.

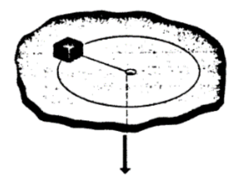
5. A rod of length L and mass 4M is pivoted about its center and allowed to rotate. Initially, it is at rest. A ball of silly putty of mass M hits the very right end of the rod and sticks to it, as shown.



a) Determine an expression for the angular speed of the rod after putty sticks to it.

b) Suppose that the silly putty instead hit the rod closer to the center than in part a). How would this affect the angular velocity after the collision? Justify your answer.

6. A puck on a frictionless air-hockey table has a mass of 0.05 kg and is attached to a cord passing through a hole in the table as shown. The puck originally revolves at a distance of 0.2 m from the hole with an angular velocity of 3 rad/s. The cord is then pulled in from below, and the cord is shortened to 0.1 m. Consider the puck as a point mass.



a) What is new angular velocity?

b) What is the amount of work done by the person who pulled the cord?

7. A large turntable of radius 2.0 m and total mass 120 kg is rotating about a vertical axis through its center, with angular velocity of 3.0 rad/s. A 100 kg crate is dropped vertically onto the turntable on its outer edge. Find the angular velocity of the turntable after the crate is dropped.

8. A dart of mass m is fired at a speed v towards a bicycle of mass $24m$ as shown on the right. The dart hits the tire at the edge and becomes embedded in it, causing the tire to rotate. The wheel has a radius R and a moment of inertia given by $I = \frac{3}{5}MR^2$.

a) Determine an expression for the initial moment of the dart above the center of mass of the wheel before the collision.

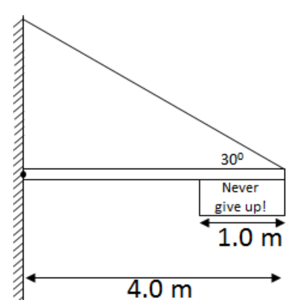
b) Determine an expression for the angular speed of the tire after the dart strikes.

c) How would the magnitude of b) change if the wheel had the same mass, but half the radius and the dart were to hit at the same speed at the edge? Justify your answer.

9. A solid cylinder of mass M = 12.0 kg and radius R = 0.20 m rolls down an inclined plane without slipping. The angle of the incline is θ = 30°.Calculate the rate of change of the angular momentum of the cylinder as it rolls.

5.6 – Statics

1. Matsuoka Shuzo hangs a motivational sign up outside an asiatic clam farm. The sign has a mass of 3.0 kg and width 1.0 m and is attached to the end of a uniform steel beam of mass 9.0 kg and length 4.0 m. The beam is attached to a fixed wall via a hinge. The beam is held up by a massless wire that is attached to the wall and makes a 30^o with the beam. The moment of inertia for a rod pivoted at one end is $\frac{1}{3}ML^2$.

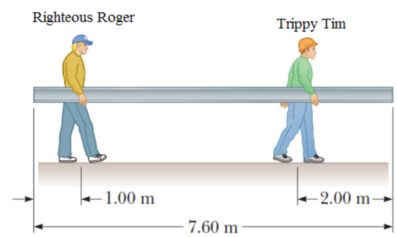


a) Calculate the tension in the wire holding up the beam.

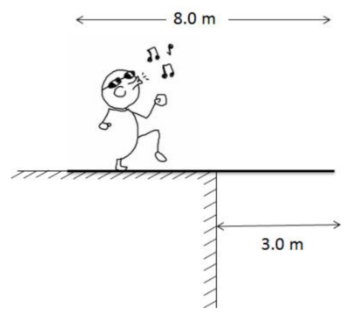
b) Calculate the total magnitude of the force of the wall on the hinge.

c) Suppose that cable is cut and there is no friction in the pin connecting the crane to the wall. Calculate the initial angular acceleration of the sign.

2. A uniform beam of length 7.60 m and weight 600 N is carried by two workers, Righteous Roger and Trippy Tim, as shown below. Determine the force that each person exerts on the beam.



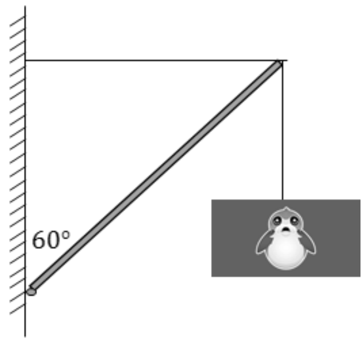
3. An 8.0-meter uniform plank of mass $10\bar{0}$ kilograms rests on the top of a building with 3.0 meters extended over the edge as shown. How far can a 75.0 kg person venture past the edge of the building before the plank begins to tip?



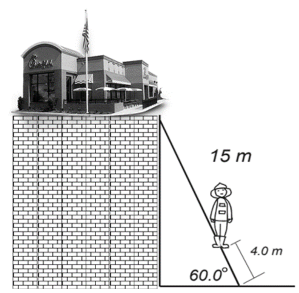
4. A 20 m long crane supported at its lower end by a pin is elevated by a horizontal cable as shown in the figure. A 60 kg Porg billboard is hung on the other side as shown. The crate itself has a mass of 30 kg. (Moment of inertia of a rod pivoted at one end; $I = \frac{1}{3}ML^2$).

a) Calculate the tension in the horizontal cable.

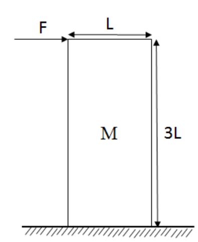
b) Suppose the crane is lowered so the angle the crane makes with the vertical is increased from 60^o. Will the tension in the support cable now increase, decrease, or remain the same?



5. A 70 kg student climbs a 15 m long ladder to buy Chik Fil-a. The ladder itself has a mass of 10 kg. The ladder is set at an angle of $\theta = 60°$ above the floor as shown and rests against a smooth wall. The student climbs 4 m up the ladder when it becomes to slip.
a) Calculate the coefficient of static friction between the ladder and floor.
b) The student wants to get closer to the Chik Fil-a at the top without the ladder slipping. Should they increase or decrease θ? Justify your answer.



6. A force F is applied at the very top of a block that is resting on a rough surface as shown. The block has a mass of M, a height of 3L, and width of L as shown. For what minimum coefficient of friction will the block tip over instead of sliding when the force is applied?



7. A wheel of mass 8 kg and radius R = 0.6 is to be pushed over a curb of height h = 0.2 m as shown. A force is applied to the wheel at its center of mass. What minimum force is needed to push the wheel over the curb?