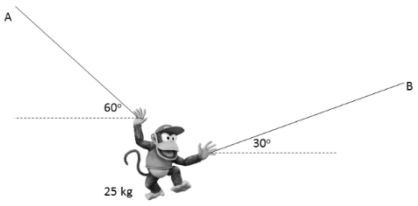
AP Physics C: Mechanics
Unit 7 Practice Exercises

7.1 – Pendulums

1. The period of a pendulum is 3.0 s at sea level.
   a) What is the length of the pendulum?
   b) Write an equation that could represent the angle θ that the pendulum makes with the vertical as a function of t. Assume it is released at an angle of θ_{max}.

2. A pendulum 1.00 m long is given an initial angular displacement of 10.00° and released from rest.
   a) What is the maximum linear acceleration of the pendulum bob?
   b) What is the maximum angular acceleration of the pendulum?
   c) What is the maximum linear velocity of the pendulum bob?
   d) What is the maximum angular velocity of the pendulum bob?

3. Diddy Kong is a monkey with a mass of 30 kg. He hangs from two vines, A and B, as shown.



   a) Which vine has more tension? Justify your answer qualitatively.
   b) Diddy Kong releases vine B and swings from vine A. Diddy Kong swings at a pace such that he completes one complete cycle every 1.5 s. Calculate the length of the vine.
   c) Using the length of the vine you found in part b), find the tension in the vine when Diddy Kong is at maximum speed.

4. A 4.0 kg mass is attached to the end of a cord, forming a pendulum. When the mass reaches the bottom of its swing, a tension of 50 N exists in the 4.0 long cord?
   a) What is the acceleration of the mass at the bottom of its swing?
   b) What is the speed of the mass at the bottom of its swing?

5. A pendulum on earth is 10 m and set into motion by releasing it when it is displaced 8^0 relative to the vertical. At what approximate time after release will it have fallen to a perfectly vertical position for the first time?

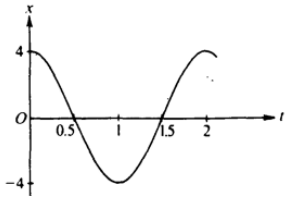
6. A thin, uniform rod of length L and mass m is pivoted about a perpendicular axis through the rod at a distance of L/7 from one end.
   a) Find the momentum of inertia about this axis.
   b) Find the period of oscillation of the rod.

7.2 – Springs

1. The position of an oscillation object varies with time according to $x = 5.0 \sin(\pi t + \frac{\pi}{6})$. What is the acceleration of the object at time t = 1.0 s?

2. The acceleration of a mass-spring system is given by $a(t) = -32\cos(4t + 2)$. Find the amplitude of the mass's motion.

3. A particle moves in simple harmonic motion represented by the graph shown.



   a) Write an equation for the position of the object as a function of time.
   b) Write an equation for the velocity of the object as a function of time.
   c) Write an equation for the acceleration of the object as a function of time.

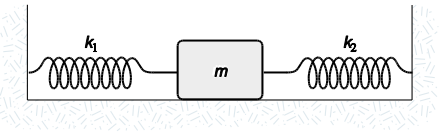
4. A mass of 25.0 kg is attached to a spring. An 80 force is applied to the spring, compressing it 20 cm from its equilibrium position. The spring is then released and allowed to oscillate. Determine an expression for the position of the spring as a function of time.

5. The motion of a particle connected to a spring is described by $x = 10 \sin \pi t$. At what time is the potential energy equal to the kinetic energy?

6. An ideal spring is fixed to a wall. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is V. Determine an expression for the force constant.

7. The equation of motion of a simple harmonic oscillator is $d^2x/dt^2 = -8x$. What is the period of this motion?

8. The mass in the figure slides on a frictionless surface. If m = 2 kg, k_1 = 800 N/m and k_2 = 500 N/m, what is the frequency of oscillation?



9. An object vibrates in simple harmonic motion with amplitude 12 cm and a frequency of 3.0 Hz. Calculate the following:

a) the maximum values of the acceleration and velocity
b) the acceleration and velocity when the coordinate is 9 cm
c) the time required to move from the equilibrium position to a point 9 cm from it.

10. In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression

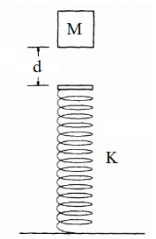
$$x(t) = .4\cos\left(2t + \frac{\pi}{3}\right)$$

a) At t = 0, find: i. the position of the particle iii its velocity iii. its acceleration.
b) Calculate the period.
c) Determine an expression for the acceleration of the particle.
d) Determine a time when the particle passes through equilibrium.
e) Determine a time when the particle is at its maximum displacement from equilibrium.
f) Determine whether the particle is speeding up at time t = 3 s.

11. A block of mass $2m$ attached to a horizontal spring of force constant k is moving with simple harmonic motion of amplitude A. At the instant it passes through its equilibrium position, a lump of clay of mass m is dropped onto the block from a very small height and sticks to it.

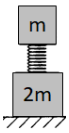
a) Find the new period and amplitude.
b) Was there a loss in mechanical energy? If so, where did it go?
c) Would the answers above be the same if the clay was dropped on the block when it was on one end of its path?

12. A block of mass $M = 0.5\ kg$ is dropped from a height $d = 2.0\ m$ above an uncompressed ideal spring, as shown. The spring has an elastic constant of $K = 40\ N/m$ and negligible mass. The block strikes the end of the spring and sticks to it.



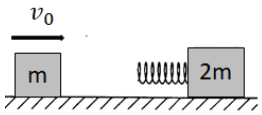
a) Calculate an expression for the speed of the block at the instant it hits the end of the spring.
b) Determine the period of the simple harmonic motion that ensues.
c) Determine the distance the spring is compressed at the instant the speed of the block is maximum.
d) Determine the amplitude of the simple harmonic motion.

13. Two masses m and 2m respectively. The block of mass 2m has an ideal massless spring attached to one side. When the block of mass m is placed on the spring as shown the spring is compressed a distance D at equilibrium



a) Determine the spring constant of the spring

b) Later the two blocks are on a frictionless, horizontal surface. The 2m block is at rest and the block of mass m is given an initial speed v_0 towards the spring.

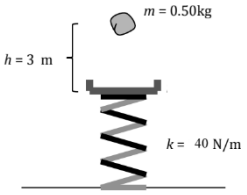


i. The spring compression is a maximum when the blocks have the same velocity. Why is this?

ii. Determine the maximum compression of the spring during the collision.

ii. Determine the velocity of the block of mass 2m after the collision.

14. A massless platform is fixed to a spring of constant k=40 N/m as shown. Anakin drops a 0.5 kg lump of sand onto the platform from a height of h = 3 m onto the platform. Taking the starting height to be y = 0 and the time of the collision to be t = 0, write an equation for y(t), the height of the sand as a function of time after landing in the platform.



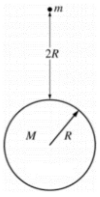
7.3 – Gravity

1. After the battle of Yavin IV, in which the rebel alliance achieved victory by destroying the insidious Death Star, the rebel alliance relocated their base to the planet Hoth, a remote ice planet unknown to the evil Galactic Empire. Of course, the secrecy of their hidden base would be short lived as an Imperial probe droid would locate the base, prompting Darth Vader's fleet to attack the planet. Oh, and Hoth happens to have three times the mass of Earth and twice the radius of Earth. What is the period of a pendulum of length L on Hoth?

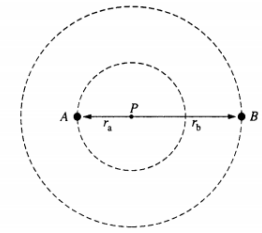
2. After their defeat at the Battle of Hoth, the rebels fled the ice world to flee from the Empire's wrath. While Han and Leia went to Cloud City to fix the Millennium Falcon's hyperdrive, Luke Skywalker went to the swampy planet of Daghaboh to learn the ways of the force from the wise Muppet, Yoda. The planet Daghaboh has the same radius as Earth, but only one-third the density. If Luke weighs 700 N on Earth, how much would he weigh on Daghaboh?

3. A projectile is launched from the surface of a planet (mass = *M*, radius = *R*). What minimum launch speed is required if the projectile is to rise to a height of 2*R* above the surface of the planet? Disregard any dissipative effects of the atmosphere.

4. A planet has a mass M, radius R, and no atmosphere. An object of mass m is located at a distance 2R above the surface of planet X, as shown. The object is released from rest and falls to the surface of the planet. What is the speed of the object just before it reaches the surface?



5. Two stars, A and B. are in circular orbits of radii r_a and r_b, respectively, about their common center of mass at point P, as shown above. Each star has the same period of revolution T.



Determine expressions for the following:

a) The centripetal acceleration of star A, T, and fundamental constants.
b) The mass of star B
c) The mass of star A
d) The moment of inertia of the two-star system about its center of mass.
e) The angular momentum of the system about the center of mass

6. A satellite orbits Mars. Assume a circular orbit with a period of 118 minutes and orbital speed of 3400 m/s. The mass of the GS is 800 kg and the radius of the orbit is 3.43×10^6 m

a) Calculate the radius of the GS orbit.

b) Calculate the mass of Mars.

c) Calculate the total mechanical energy of the GS in this orbit.

d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period? Justify your answer.

e) In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

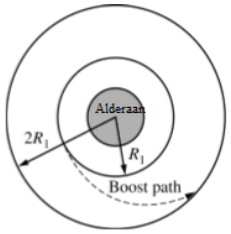
7. A satellite of mass m is in a stable circular orbit around Alderaan at a distance R_1 from the center of Alderaan. The mass of Alderaan is M_N. Alderaan is a peaceful planet; they have no weapons.

a) Derive an expression for the following in terms of the given variables and fundamental constants.

i. The orbital speed of the satellite.

ii. The total energy of the satellite in this orbit, assuming gravitational potential energy to be zero at an infinite distance from the center of Alderaan.

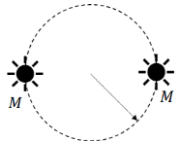
b) The satellite's booster rocket fires and lifts the satellite to a higher circular orbit of radius $2R_1$. The satellite follows the path shown below, a moving a total distance S during the orbital change. The component of the rocket's force parallel to the path is given by $F = F_0(1 - \frac{x}{S})$, where x is the variable distance traveled along the path at any moment.



i. Derive an expression for the total work done on the satellite by the force F while changing orbits in terms of F_0 and R_1 if $S = R_1$.

ii. How will the orbital period of the satellite in its new orbit compare to the orbit from part a)?

8. Two stars each of mass M form a binary star system such that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.



a) Use Newton's laws of motion and gravitation to find an expression for the speed of each star.

b) Express the total energy E of the binary star system in terms of R, G, and M.

c) Suppose instead, one of the stars had a mass 2M. Find the ratio of the speeds, v_{2M}/v_M.

9. A spherical, nonrotating Death Star has a radius R and a uniform density ρ throughout its volume. Suppose a narrow tunnel were drilled through the star along one of its diameters, as shown in the figure, in which Palpatine, of mass m could move freely under the influence of gravity after Darth Vader yeets Palpatine down the tunnel. Let r be the distance of the mass from the center of the star. Derive an expression for the period of the simple harmonic motion of the mass.